A new performance measure for cell formation problems considering alternative routing and operation sequences

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***Abstract*— In this paper, a new performance measure for cell formation problems considering alternative routing and operation sequences is presented. Due to the combinatorial nature of cell formation problem, a simulated annealing-based approach has been proposed to address this issue. A test instance from the literature is employed to illustrate the effectiveness of the proposed approach. Computational results from test problem show that our proposed performance measure and solution approach are both effective and efficient. When compared to the mathematical programming approach, which takes more than 2.7 hours to solve the test instance, the proposed algorithm can produce optimal solution for the same test instance in less than 1 second. Thus, it deserves more attentions and can be treated as an alternative performance measure for cell formation problems on account of alternative routing and operation sequences.**

***Keywords*—Cell formation; Operation sequence; Grouping efficacy; Performance measure**

1. INTRODUCTION

Group Technology (GT) is a manufacturing philosophy in which similar components/parts are identified and grouped into part families, machines are grouped into machine cells in order to make full use of their similarities in manufacturing and design. The implementation of cellular manufacturing has been reported to result in significant benefits such as reductions in set-up times, work-in-progress inventory, throughput times and material handling costs, simplified scheduling and improved quality [1]. Many models and solution approaches have been developed to identify machine cells and part families; but whatever the method used, one should choose the method that is based on some measures to indicate the goodness of the solution.

Table 1 is the summary of related measures for cell formation problems. From this table, we can see that most current performance measures for cell formation problems are inappropriate for evaluating the cell formation plans generated under production environments that considers alternative routing and operation sequences.

TABLE 1 Summary of related measures

Measure	Name	Reference	BD	OS	AR
1	Grouping efficiency	[2]	✓		
2	Global efficiency	[3]		✓	
3	Group efficiency	[3]		✓	
4	GT efficiency	[3]		✓	
5	Grouping efficacy	[4]	✓		
6	Group capability index	[5]	✓		
7	Grouping index	[6]	✓		
8	Quality index	[7]	✓		
9	Generalized grouping efficiency	[8]	✓		✓
10	Bond efficiency(BE)	[9]		✓	
11	Double weight grouping efficacy	[10]	✓		
12	Generalized grouping efficacy	Proposed measure	✓	✓	✓

BD: binary data, OS: operation sequence, AR: alternative routing

For this reason, we propose a new performance measure where alternative routing and sequence data exist. This new performance measure is then embedded into a simulated-annealing (SA) based algorithm as the decision objective in order to derive the best grouping plan. An instance is used to demonstrate the effectiveness of the new performance measure and the corresponding SA-based methodology. The results derived are encouraging as compared with the results of other performance measures.

The remainder of the paper is organized as follows. Section 2 gives a background of cell formation problems. Section 3 discusses the details the proposed performance measure, and Section 4 presents the SA-based methodology for forming cells. The latter section also shows the computational results of the test problems. Finally, the conclusions are presented in Section 6.

2. CELL FORMATION PROBLEMS

In a simple CFP, cell formation in a given 0-1 machine-part incidence matrix involves rearrangement of rows and columns of the matrix to create part families and machines cells, in which the cellular movement can be minimized and the utilization of the machines within a cell can be maximized. Two matrices shown in Figure 1 are used to illustrate the concept. Fig. 1(a) is an initial matrix where no blocks can be observed directly. After rearrangement of rows and columns, two blocks can be obtained along the diagonal of the solution matrix in Fig. 1(b). For those 1's outside the diagonal blocks, they are called "exceptional elements"; while those 0's inside the diagonal blocks are called "voids".

(a)

Machine \ Part	P1	P2	P3	P4	P5
M1	0	1	0	1	0
M2	0	0	1	0	1
M3	1	1	0	1	0
M4	1	0	1	0	1
M5	0	1	0	0	0

(b)

Machine \ Part	P1	P3	P5	P2	P4
M2	0	1	1	0	0
M4	1	1	1	0	0
M1	0	0	0	1	1
M3	1	0	0	1	1
M5	0	0	0	1	0

Fig. 1 Rearrangement of rows and columns of matrix to create cells: (a) initial matrix and (b) matrix after rearrangement.

When parts have alternative process routings (APR) is called the generalized CFP. Such as the case shown in Table 2, part #1 has two process routings R1 and R2. While introducing APR to CFP, the grouping of parts can be more effective due to the flexibility of the routes; however, it leads to a more complex problem than the simple CFP. Under this circumstance, not only the formation of part families and machine cells must be determined but also the selection of routings for each part has to be determined to achieve decision objectives such as the minimization of intercellular movement. For instance, Fig. 2 provides a feasible solution to the sample problem of Table 2 which has three cells with machine groupings for each cell as Cell 1: (M3, M7); Cell 2: (M2, M4, M6); and Cell 3: (M1, M5, M8).

TABLE 2 Initial machine-part matrix where alternative process routings are allowed

PN	P1		P2		P3		P4		P5		P6	
RN	R1	R2	R1	R2	R1	R2	R1	R2	R1	R2	R1	R2
PV	150		95		130		80		95		135	
M1	1*			1		1			1			1
M2	2	2			1				2		1	2
M3							1	1	3	1		
M4	3	1									2	3
M5				2		2						
M6		3									3	
M7			1		2	3	2	2		2		
M8			2	3						3		

PN: Part Number; PV: Production Volume; RN: Routing Number; * Process Sequence

PN	P4	P5	P1	P6	P2	P3
RN	R2	R2	R2	R1	R2	R2
PV	80	95	150	135	95	130
M3	1	1				
M7	2	2				3
M2			2	1		
M4			1	2		
M6			3	3		
M1					1	1
M5					2	2
M8		3			3	

Fig. 2 Final machine-part matrix of Table 2

3. PROPOSED PERFORMANCE MEASURE

Most of the models and solution approaches developed to determine machine cells and part families use the machine-component incidence matrix (MCIM), which is composed of binary values, when constructing their computational logic. However, MCIM only indicates whether certain components/parts visit certain machines. Moreover, these methods usually have the following deficiencies, as pointed out by some researchers ([3] [9] [11]):

1. Failure to address the issue of production sequence..
2. Failure to address the issue of non-consecutive operations on the same machine.
3. Failure to address the issue of product volume.

For this reason, this study integrates the concerns of both grouping efficacy and inter-cell movements presents a new measure called 'Generalized Grouping Efficacy (GGE)' for cell formation problems on account of alternative routing and operation sequences.

The proposed performance measure GGE is shown in Eq. (1) below.

$$GGE = \frac{\Gamma}{1 + \left(\frac{ICM}{N_{tf}}\right)} \tag{1}$$

where Γ is the grouping efficacy; *ICM* is the maximum number of inter-cell movements possible; N_{tf} is the actual number of inter-cell movements required by the system. Grouping efficacy ranges from 1 to 0, with 1 being the perfect grouping.

The design of GGE integrates the concerns of both grouping efficacy and inter-cell movements. As compared to the grouping efficacy, GGE offers the following features:

1. It requires less input information/data in calculating the performance measure;
2. There is no need to indicate the values for any parameters of the performance measure, such as the weighting factor *q* in bond efficiency;
3. It is obtained through a direct revision to a widely known and adopted measure for simple cell formation problems, i.e., the grouping efficacy.
4. It not only suitable for machine-component incidence matrix with alternative routing, operation sequence, and production volume, but also suitable for machine-component incidence matrix with binary data (i.e. let *ICM*=0).

4. MATHEMATICAL MODEL

The decision objective is mainly to solve the cell formation in terms of maximizing generalized grouping efficacy. The 0-1 integer programming model is given below, and the notations are introduced first.

A. *Notations*

- a : Index for operations which belongs to part i along route j (a=1,..., K_{ij})
- b : Index for position number (or index for sequence of machine)
- i : Index for parts (i=1,..., p)
- j : Index for routings which belongs to part i (j=1,..., Q_i)
- k : Index for machines (k=1,..., m)
- l : Index for manufacturing cells (l=1,..., NC)
- p : Number of parts
- Q_i : Number of routings for part i
- m : Number of machines
- r : Number of routings
- NC : Number of cells
- L_m : Minimum number of machines in each cell
- U_m : Maximum number of machines in each cell
- K_{ij} : Number of operations in routing j of part i
- r_i : Best routing selection for part i
- N_{tf} : Total number of flows
- $u_{ij}^{(a)}$: Index for machines which belongs to the a-th operation of part i along route j
- V_i : Production volume for part i
- Γ : Grouping efficacy
- e : The total operations in the incidence matrix
- e_0 : The total number of exceptional elements
- e_v : The total number of voids
- X_{il} : 1, if part i locates in cell l; 0, otherwise
- Y_{kl} : 1, if machine k locates in cell l; 0, otherwise
- Z_{ij} : 1, if routing j of part i selected; 0, otherwise

B. *Mathematical model*

Maximizing Generalized grouping efficacy (2)

$$(GGE) = \frac{\Gamma}{1 + \left(\frac{ICM}{N_{tf}}\right)}$$

Subject to:

$$\Gamma = \frac{e - e_0}{e + e_v} \tag{3}$$

$$e = \sum_{i=1}^{p}\sum_{j=1}^{Q_i}\sum_{k=1}^{m} a_{ik} Z_{ij} \tag{4}$$

$$e_v = \sum_{l=1}^{NC}\sum_{i=1}^{p}\sum_{j=1}^{Q_i}\sum_{k=1}^{m} (1 - a_{ik}) X_{il} Y_{kl} Z_{ij} \tag{5}$$

$$e_0 = e - \sum_{l=1}^{NC}\sum_{i=1}^{p}\sum_{j=1}^{Q_i}\sum_{k=1}^{m} a_{ik} X_{il} Y_{kl} Z_{ij} \tag{6}$$

$$ICM = \sum_{i=1}^{p}\sum_{j=1}^{Q_i}\sum_{a=1}^{K_{ij}-1}\sum_{l=1}^{NC} Z_{ij} Y_{(u_{ij}^{(a)})l}(1 - Y_{(u_{ij}^{(a)})l} Y_{(u_{ij}^{(a+1)})l}) V_i \quad (7)$$

$$N_{tf} = \sum_{i=1}^{p}\sum_{j=1}^{Q_i} Z_{ij}(K_{ij} - 1)V_i \quad (8)$$

$$\sum_{j=1}^{Q_i} Z_{ij} = 1 \ , \ \forall i \quad (9)$$

$$L_m \le \sum_{k=1}^{m} Y_{kl} \le U_m \ , \ \forall l \quad (10)$$

$$\sum_{l=1}^{NC} Y_{kl} = 1 \ , \forall k \quad (11)$$

$$\sum_{l=1}^{NC} X_{il} = 1 \ , \forall i \quad (12)$$

$$X_{il},\ Y_{kl},\ Z_{ij} \in \{0,\ 1\},\ \ \forall i, j, k, l \quad (13)$$

In the above model, Eq. (2) is the objective function, which is to minimizing inter-cell flows and maximizing grouping efficacy. Eq. (3) shows the calculation of the grouping efficacy. Eqs. (4), (5), and (6) show the calculation of the total operations in the machine-part incidence matrix, the total number of voids, and the total number of exceptional elements, respectively. Eq. (7) shows the calculation of the total inter-cell movements required by the system. Eq. (8) shows the calculation of the maximum number of inter-cell travels possible in the system. Eq. (9) indicates that just a single process routing will be assigned to each part. Eq. (10) indicates that upper and lower limit of the cell size. Eq. (11) provides a restriction that each machine will be assigned to exactly one cell, while Eq. (12) provides a restriction that each part will be assigned to exactly one cell. Eq. (13) indicates that X_{il}, Y_{kl} and Z_{ij} are 0–1 binary decision variables.

The objective function is a non-linear form. Thus, developing good heuristic approaches is more appropriate than the exact method in terms of solving efficiency, especially for large-sized problems. A SA based heuristic algorithm is proposed and discussed in the next section.

5. PROPOSED ALGORITHM

The main disadvantages of SA are as follows: (1) high execution time, (2) ease of being trapped to local minima if the cooling speed is too fast or the initial temperature is not high enough, and (3) difficulty of obtaining a globally optimum solution if the search cannot reach the equilibrium state at each temperature. In this study, two types of mechanisms, the insertion-move and the mutation strategy of GA, are utilized to construct a hybrid SA method called HSAM to address these issues. Both mechanisms play different roles in the process of solution improvement. We use insertion-move as a primary tool for finding better neighborhood solution, while employing mutation strategy to increase the probability of finding more "diversified" solutions to bring the searching process to a new and unexplored solution space. The proposed procedure HSAM is described in detail below.

A. *Notations*

α	:	Cooling rate
counter_mut	:	Number of times the mutation strategy has been implemented
C^*	:	Optimal number of cells
f(S)	:	Value of object function in solution *S*
L	:	Markov chain length
NC	:	Number of cells
N^C	:	Set of solutions without violating cell cardinality constraints
Stag_check	:	Maximum number of solution has not been improved
S^0	:	Initial solution
S	:	Current solution
S^N	:	Neighborhood solution
S^*	:	Incumbent solution of current cell size
S^{**}	:	Best solution found so far
T_0	:	Initial temperature
T_f	:	Final temperature

B. *Algorithm HSAM*

Step 1. Generate an initial solution S^0. Set NC=2, $S^{**} = S^* = S^0$, $NC^* = NC$

Step 2. Initialization: Let *counter_MC* = 0, $T = T_0$, $S \leftarrow S^0$, $S^* \leftarrow S^0$.

Step 3. If *counter_MC* < *L*, then repeat Steps 3.1 to 3.5; otherwise, go to Step 4.

Step 3.1. If *counter_mut* ≥ *mut_check*, then apply the **mutation strategy** to generate a new current solution *S* and let *counter_mut* = 0.

Step 3.2. Generate a best solution S^N $\left(S^N \in N^C\right)$ in the neighborhood of S by performing the **insertion-move operation**.

Step 3.3. Compute $\Delta = f\left(S^N\right) - f\left(S\right)$. If (($\Delta > 0$) or ($e^{\Delta/T} > r \in U(0,1)$)), then let

$S \leftarrow S^N$, *counter_mut* = 0; otherwise, *counter_mut* = *counter_mut* + 1.

Step 3.4. If ($f(S^N) > f(S^*)$), then let $S^* \leftarrow S^N$.

Step 3.5. Let *counter_MC*= *counter_MC* + 1, go to Step 3.

Step 4. If $T \le T_f$, then go to Step 5; otherwise, $T = T \times \alpha$, *counter_MC* = 0, go to Step 3.

Step 5. If $f\left(S^*\right) > f\left(S^{**}\right)$, the $S^{**} = S^*$, $NC^* = NC$, $NC = NC+1$, go to Step 2; otherwise report the best solutions so far, and stop the algorithm.

Note that the algorithm starts from an initial solution in Step 1, after which all algorithmic parameters and counters are initialized in Step 2. As long as the value of *counter_mut* is smaller than *mut_check*, a new neighbourhood solution is generated through the insertion-move in Step 3.2; otherwise, mutation strategy is applied to generate a new solution with higher degree of diversification in Step 3.1. If the newly generated neighbourhood solution is better than the current solution or the probability function ($e^{\Delta/T}$ is great than a random number *r*), a replacement is made and the *counter_mut* will be set to 0 in Step 3.3; otherwise, the *counter_mut* is increased by 1. The incumbent solution will be updated in Step 3.4 if the newly generated neighbourhood solution results in a better objective value. Step 3 will be repeated *L* times at each temperature to reach the thermal equilibrium. Parameter *T* is gradually decreased by a cooling function and the solution process repeats until the stopping criteria in Step 4 is met.

6. RESEARCH RESULTS

To validate the quality of the solutions provided by the proposed approach, one test instance, as shown in Table 3, is solved in this research. It is prepared by adding self-created data such as operation sequences (OS) and production volumes (PV) to test instances chosen from literature [12].

The problem size (machine×part×routing) of the test instance is 10×10×24. HSAM is coded in C++ and implemented on an Intel(R) 2.40 GHz personal computer with 3.24 GB RAM.

Table 4 shows a comparison between the computational results for both the maximizing grouping efficacy (HSAM1) and maximizing generalized grouping efficacy (HSAM2). In the test problem, HSAM1 and HSAM2 produce the same grouping efficacy; however, the HSAM2 produces better results than those by the HSAM1 in the test problem in terms of ICM and GGE.

The mathematical model described in Section 4 is solved using Lingo 8.0 software. The Lingo solver status is shown in Fig. 3. The optimal solution (0.766648) can be obtained in less than 9702 s (2.7 h). In contrast, our proposed HSAM2 is able to find the optimal solution in 0.34 second, thus illustrating the superiority of HSAM in solution efficiency. Similarly, we believe this superiority will be even more significant as problem size increases.

The final machine-part matrixes for the test instance obtained by HSAM1 and HSAM2 are presented in figures 4 and 5. The solutions include the part families and machine cells.

TABLE 1 Production data

PN	P1			P2		P3			P4		P5		P6			P7			P8		P9		P10	
RN	1	2	3	1	2	1	2	3	1	2	1	2	1	2	3	1	2	3	1	2	1	2	1	2
PV	5			5		5			42		19		20			37			40		32		8	
M1	1	.	.	.	1	.	.	2	.	.	1	.	1	.	.	.	1	.	1	1	.	.	1	.
M2	2	.	.	.	.	2	1	.	.	.	.	1	2	.	.	1	2	.	.	.	.	.	.	.
M3		.	1	.	.	1	.	1	1	1	.	.	3	1	.	.	.	.	.	.	1	1	2	1
M4	3	1	2	.	.	.	.	.	.	.	.	.	.	.	.	2	3	1	.	.	.	.	.	.
M5	.	.	.	.	2	.	.	.	.	.	2	2	.	.	1	.	.	.	2	.	.	.	.	.
M6	.	2	.	.	.	.	.	.	.	.	.	.	.	.	2	3	.	.	.	.	.	.	.	.
M7	.	.	.	1	.	3	2	.	2	2	.	.	.	2	3	.	.	2	3	.	2	2	.	2
M8	.	.	.	2	.	.	3	.	.	3	.	.	.	3	.	.	.	3	.	.	3	.	3	3
M9	.	.	.	.	3	.	.	.	3	.	.	3	.	.	.	.	.	.	4	2	.	3	.	.
M10	.	.	3	3	4	.	.	3	.	.	3	4	.	.	.	.	.	.	5	3	4	.	.	.

TABLE 4 Comparison of Lingo and our WFACF

Method	***NC***	**void**	***ICM***	**Efficacy (%)**	**GGE (%)**	**CPU (s)**
[1]HSAM1	3	4	42	82.86	75.9	0.42
[2]HSAM2	3	4	37	82.86	76.66	0.34
Lingo (Global)	3	4	37	82.86	76.66	9702

[1]Maximizing grouping efficacy

[2]Maximizing generalized grouping efficacy(GGE)

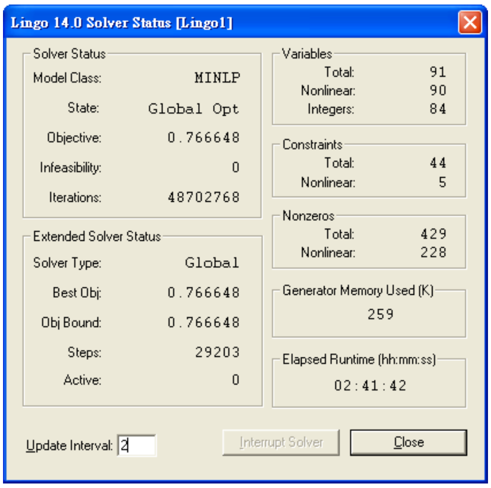

Fig. 3 Lingo solver status for test instance

PN	P3	P4	P6	P9	P10	P1	P7	P2	P5	P8
RN	1	2	2	1	2	2	1	2	1	2
PV	5	42	20	32	8	5	37	5	19	40
M3	1	1	1	1	1	.	.	.	.	.
M7	3	2	2	2	2	.	.	.	.	.
M8	.	3	3	3	3	.	.	.	.	.
M2	2	.	.	.	.	.	1	.	.	.
M4	.	.	.	.	.	1	2	.	.	.
M6	.	.	.	.	.	2	3	.	.	.
M1	.	.	.	.	.	.	.	1	1	1
M5	.	.	.	.	.	.	.	2	2	.
M9	.	.	.	.	.	.	.	3	.	2
M10	.	.	.	4	.	.	.	4	3	3

Fig. 4 Final machine-part matrix with maximizing grouping efficacy (HSAM1)

PN	P3	P4	P6	P9	P10	P1	P7	P2	P5	P8
RN	2	2	2	1	2	2	1	2	1	2
PV	5	42	20	32	8	5	37	5	19	40
M3	.	1	1	1	1	.	.	.	.	.
M7	2	2	2	2	2	.	.	.	.	.
M8	3	3	3	3	3	.	.	.	.	.
M2	1	.	.	.	.	.	1	.	.	.
M4	.	.	.	.	.	1	2	.	.	.
M6	.	.	.	.	.	2	3	.	.	.
M1	.	.	.	.	.	.	.	1	1	1
M5	.	.	.	.	.	.	.	2	2	.
M9	.	.	.	.	.	.	.	3	.	2
M10	.	.	.	4	.	.	.	4	3	3

Fig. 5 Final machine-part matrix with maximizing generalized grouping efficacy (HSAM2)

6. CONCLUSIONS

Very limited amount of performance measures have simultaneously considered the issues of production sequence and alternative process routings in CFP so far. Accounting for these factors makes the CFP complex but more realistic. In this paper, a new performance measure for cell formation problems considering alternative routing and operation sequences is presented. Due to the combinatorial nature of this model, a SA based algorithm has been designed for solving this problem.

The results of the proposed method are compared with the optimal solutions obtained by the LINGO 8.0 software. The comparisons show that the proposed method offers good solutions for the CFP considering production sequence and alternative process routings. More test problems from the open literature are to be tested in the near future to fully confirm the efficiency and effectiveness of the proposed approach.

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