A Novel Least-Squares Approach for Estimation of OFDM Channel Frequency Response

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***Abstract*—A novel least-squares (LS) scheme to estimate the channel frequency response (CFR) for orthogonal frequency division multiplexing (OFDM) over slow fading channels is presented. We take the advantage of slow fading to use repeated OFDM training blocks. This not only can facilitate the LS formulation but also improve the channel estimation performance as compared to the conventional one block LS channel estimation. In addition, we show analytically as well as by simulations that a constant unit amplitude sequence provides the optimal training sequence.**

Keywords—**Orthogonal frequency division multiplexing, channel estimation, least-squares estimation, channel frequency response, optimal training sequence**

1. INTRODUCTION

For orthogonal frequency division multiplexing (OFDM) systems, accurate data detection requires accurate estimate of the channel frequency response (CFR) of all subcarriers [1]. In the literature, numerous OFDM channel estimation schemes have been proposed [2]-[17]. For fast fading channels, dynamic channel estimation with pilot data arrangement is usually employed [3], [4], [7], [13], [15]. When channel fading is slow, the channel estimate periodically obtained from one initial OFDM training block can be used for data detection in the following periods of OFDM data blocks [5], [6], [8]-[12], [14], [16], [17]. In this paper, we shall address slow fading channels. That is, fading is assumed to remain essentially constant over a period of many OFDM blocks (many researchers use the term 'OFDM symbols'). Popular estimation methods include minimum mean square error (MMSE) estimation [2], [4], [5], [16], maximum-likelihood (ML) estimation [16], [17], and LS estimation [2], [4], [5], [8], [12], [14], [15]. The MMSE channel estimation is computationally complex and requires channel correlation as well as noise power information [2], [4]. For ML estimation, one needs to form a log-likelihood function in terms of the estimated parameters. For LS estimation, one needs to form an overdetermined system of linear simultaneous equations for the estimated parameters. It is important to note that, for both ML and LS estimations, the parameters to be estimated must be independent of each other. In a frequency-selective channel, the time-domain channel taps, or the tap channel impulse responses (CIRs), are uncorrelated [1]. But in an OFDM system operated over frequency-selective channels, the subcarrier channel frequency responses (CFRs) are correlated. This is because the subcarrier CFRs are the discrete Fourier transforms (DFTs) of CIRs. Therefore, when performing ML or LS channel estimations, the estimated parameters are usually the CIRs. After finding the CIR estimators, the CFR estimators can be readily obtained through DFT. When using ML and LS methods, most previously cited works use one OFDM training block to perform channel estimations. We note here that the ML estimate possesses the invariance property, i.e., the ML estimate of a function of a parameter is simply that function of the ML estimate of that parameter, while the LS estimate does not have this privilege. Therefore strictly speaking, the CFR estimator that is obtained from the DFT of (thus a function of) an LS CIR estimator should not be called an LS CFR estimator.

In this paper, we scheme an LS method to directly obtain the CFR estimators of OFDM systems operated in slow fading channels without first finding the CIR estimators. We stress that the method we propose here can only be applied to slow fading channels, i.e., channel fading is assumed to remain essentially unchanged over several OFDM blocks. As a result, we can use repeated OFDM training blocks to enable the formulation of an overdetermined system in terms of a single subcarrier CFR parameter. There, we need not worry about correlations between different subcarriers.

The rest of the paper is organized as follows: Section II formulates the overdetermined system of linear simultaneous equations in terms of a subcarrier CFR using repeated OFDM blocks for an OFDM system operated in slow fading channels. From this overdetermined system, the subcarrier CFR estimators are obtained. Section III derives the theoretical expression for the MSE of the LS CFR estimators. Meanwhile, we show that a constant amplitude sequence with the amplitude having unit magnitude is an optimal training sequence for our LS channel estimation. Then, Section IV presents a simulation result compared to theoretical prediction. Finally, Section V gives the conclusion.

2. THE NOVEL LS CFR ESTIMATION ALGORITHM

Consider an *N*-point OFDM system. Denote the transmitted baseband data symbol for the *k*th subcarrier by X_k, $k = 0,1,...,N-1$. Let the *k*th subcarrier CFR be H_k. Suppose we send *M* identical OFDM blocks for training purpose. Then, assuming timing and frequency synchronizations have been completed, the demodulated output for *M* repeated OFDM blocks at the receiver can be written in vector form as

$$\mathbf{R}_k = \mathbf{X}_k H_k + \mathbf{W}_k, \quad k = 0,1,...,N-1, \quad (1)$$

where $\mathbf{R}_k = [R_{1,k}, R_{2,k}, ...R_{M,k}]^T$ and $\mathbf{W}_k = [W_{1,k}, W_{2,k}, ..., W_{M,k}]^T$ are respectively the received noisy signal vector and the additive white Gaussian noise (AWGN) vector for *M* OFDM blocks with *T* denoting transposition. The noise components $\{ W_{m,k},\ m = 1,2,...,M \}$ are independent, identically distributed random variables with zero mean and variance σ_W^2.

Using the pseudo-inverse formula, we readily obtain the LS estimate for CFR as

$$\hat{H}_k = (\mathbf{X}_k^H \mathbf{X}_k)^{-1} \mathbf{X}_k^H \mathbf{R}_k = \frac{1}{MX_k} \sum_{m=1}^{M} R_{m,k}, \quad k = 0,1,...,N-1. \quad (2)$$

If only one OFDM is used, $M = 1$ and (2) reduces to (dropping the subscript *m*)

$$\hat{H}_k = \frac{R_k}{X_k} \quad (3)$$

This is simply the LS CFR estimate using a single training block as given in [2].

We note here that the use of *M* repeated OFDM blocks requires that channel fading remains unchanged at least over *M* OFDM blocks (slow quasi-static fading). This implies that the maximum Doppler frequency must satisfy $f_M = \upsilon f_c / c << 1/T$ corresponding to a mobile speed $\upsilon << c/(f_c T)$, where *c* is the speed of light, *T* is one OFDM block length in seconds, and f_c is the carrier frequency in Hz. Using an 802.11a standard with $f_c = 5$ GHz and $\Delta f = 1/T = 312.5$ kHz, this requires $\upsilon << 67{,}500$ km/hr. Apparently, this requirement for slow quasi-static fading is easily met in practice. For example, if $\upsilon = 60$ km/hr, then $f_M \approx 10^{-3}/T$, it is thus reasonable to assume fading to remain unchanged over several hundred OFDM blocks.

3. ESTIMATOR PERFORMANCE

We can rewrite (2) as

$$\hat{H}_k = \frac{1}{MX_k} \sum_{m=1}^{M} (H_k X_k + W_{m,k}) = H_k + \frac{1}{MX_k} \sum_{m=1}^{M} W_{m,k}, \quad (4)$$

whence, the MSE of $\hat{H}_k$ can be readily calculated as

$$\sigma_{\hat{H}_k}^2 = E[|\hat{H}_k - H_k|^2] = \frac{1}{M^2 |X_k|^2} \sum_{m=1}^{M} E[|W_{m,k}|^2] = \frac{\sigma_W^2}{M|X_k|^2}. \quad (5)$$

From (5), it is apparent that using repeated training blocks improves the estimation performance over a single training block.

We further normalize the signal power as

$$P_X = \frac{1}{N} \sum_{k=0}^{N-1} |X_k|^2 = 1. \quad (6)$$

Then, define the average overall estimator MSE as

$$\sigma^2_{\hat{\mathbf{H}}_k} = \frac{1}{N}\sum_{k=0}^{N-1}\sigma^2_{\hat{H}_k} = \frac{\sigma^2_W}{NM}\sum_{k=0}^{N-1}\frac{1}{|X_k|^2}. \quad (7)$$

We can minimize $\sigma^2_{\hat{\mathbf{H}}_k}$ subject to the constraint of (6) by using the Lagrange method. We readily find

$$\lambda = \frac{N\sigma^2_W}{M}, \quad (8a)$$

$$|X_k| = 1. \quad (8b)$$

Using (8b), the minimum average estimator MSE can be found as

$$\sigma^2_{\hat{\mathbf{H}}_k \min} = \frac{\sigma^2_W}{M}. \quad (9)$$

We conclude that the optimal training sequence for our LS CFR estimation is a sequence with unit constant amplitude. The polyphase sequences given by Chu [18] are good candidates. The Chu sequences can avoid the large peak-to-average power ratio problem. They have constant magnitudes both in the time and frequency domain, and possess a desirable periodic autocorrelation of the Kronecker delta function [18]. In our simulations, we have chosen the Chu sequence given by $X_k = e^{j\pi k^2/N}$.

4. NUMERICAL RESULTS

For simulations, we take a frequency-selective channel with exponential power profile. We assume a frequency-selective Rayleigh fading channel of dispersion length 3. Also, the channel power is normalized to unity. The OFDM system is assumed to have $N = 64$ subcarriers. We use 3 repeated OFDM training blocks with a random 16-QAM sequence as well as a Chu sequence specified earlier. For fair comparison, we also normalize the power of the QAM sequence. The simulated MSEs of our CFR estimators vs. signal-to-noise ratio (SNR) are obtained in Fig. 1 by averaging over 2,000 runs. Also incorporated in the figure are theoretical MSEs given by (7) and (9). Note that, since the signal power has been normalized, the SNR is simply given by $1/\sigma^2_W$. From Fig. 1, we see that the simulated results well agree with the theoretical predictions. As expected, the performance given by the Chu sequence is far superior to the random QAM sequence. For a given MSE, the Chu sequence has a 20 dB SNR gain over the QAM sequence.

5. CONCLUSIONS

We present a least-squares estimation scheme for the channel frequency responses of OFDM systems using repeated OFDM training blocks for slow fading channels. We also show that a unit constant amplitude sequence provides the optimal training sequence for our least-squares CFR estimation. Results obtained by simulations show excellent agreement with theoretical predictions.

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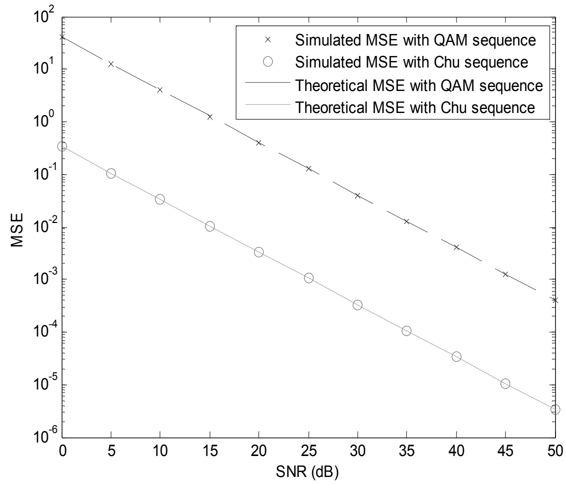

Fig. 1 MSE of LS CFR estimator vs. SNR. 3 repeated OFDM blocks.