FIR Bandpass Digital Filter Design with Transformation Method

Yang-Lung Tai
Department of Information Technology and Communication, Tungnan University
N0. 152, Sec. 3, Peishen Rd., ShenKeng, Taipei, 222, Taiwan, R.O.C.
yltai@mail.tnu.edu.tw

Abstract- A novel bandpass function is proposed as the prescribed subfilter for the design of an FIR bandpass filter. The overall filter is determined by the prototype filter and the prescribed subfilter through the frequency transformation. The implementation of the prescribed subfilter is multiplierless. The prototype filter can be designed by the standard FIR design program. The number of multipliers required to implement the overall filter is equal to that required by the prototype filter. It is less than that designed by the direct form minimax method.

Keywords: FIR digital filter, prototype filter, subfilter, transformation method, bandpass, multiplierless

I. INTRODUCTION

Finite impulse response (FIR) filters have been employed in a wide range of applications such as digital audio, video, and mobile telephony. But, to implement the digital FIR filter requires considerably more arithmetic operations and hardware components than that required by the IIR equivalents. During the past years, many design methodologies have been proposed to reduce the realization complexity of FIR filters. One method to reduce the computational and hardware complexity is to design the filter by multiple use of a number of identical subfilter with the aid of a few additional adders and multipliers. This method is proposed by Kaiser and Hamming in the paper [1] and is extended in the paper [2]. The method proposed in the paper [3] is to design the filter by the tapped cascaded interconnections of identical subfilters with relatively relaxed frequency responses. This results in a desired filter with superior characteristics. This approach has been extended and generalized in the paper [4-5]. This design technique is to decompose the desired overall filter into a prototype filter and some subfilters through the frequency transformation. The passband and stopband edges of the subfilter are the same as that of the desired filter. The passband ripple and the stopband attenuation of the prototype filter are the same as that of the desired overall filter. Many design schemes are discussed for the different selection of the prototype filter and subfilter. The computational and hardware complexity of the overall filter can be determined by the relaxed ripple and attenuation of the subfilter and the relaxed transition band of the prototype filter. The zero–phase frequency response $H(e^{jw})$ of the overall filter can be expressed as

$$H(e^{jw}) = \sum_{n=0}^{N} a(n)[H_M(e^{jw})]^n. \tag{1a}$$

where

$$H_M(e^{jw}) = \sum_{r=0}^{M} h_{sub}(r)\cos(nw). \tag{1b}$$

The transfer function of $H_M(z)$ is a linear-phase filter of length 2M+1. The subfilter is determined by the frequency transformation. The a(n) coefficients are determined by the prototype filter.

The advantage of the filter designed by this method [4] is that the frequency response of the overall filter can be controlled by changing the tap coefficients of the prototype filter. Another advantage is that the filter's modular structure gives the ability to reuse a subfilter to save area by means of scheduling and resource sharing for lower frequency applications.

In the paper [6], it is to extend the method [4-5] to design a filter with the prescribed subfilter. The proposed subfilter is multiplierless. It results that the number of multipliers and the multiplication rate of the overall filter are small.

A modified transformation technique is also developed. The lowpass filter and high pass filter design are discussed. In the paper [7], the design of subfilters using identical subfilters of even length is discussed. However, the design of a filter with the multiplierless bandpass function and the transformation method is not developed in the past papers.

In this paper, a new bandpass function is developed. It is efficient for the design of a bandpass filter by the transformation method. The proposed bandpass subfilter function is multiplierless. It results that the number of multipliers and the multiplication rate of the overall filter is reduced. It is less than that designed by the standard FIR design program.

II. GENERAL TRANSFORMATION

A zero-phase FIR filter transfer function H(z) of length 2N+1 can be written as

$$H(Z)=\sum_{n=0}^{N}h(n)(\frac{Z^{n}+Z^{-n}}{2}). \qquad (2)$$

As we define $Z=e^{j\Omega}$, the frequency response of the filter can be written in equivalent form as

$$H(e^{j\Omega})=\sum_{n=0}^{N}h(n)\cos(n\Omega). \qquad (3)$$

Using the equivalence

$$\cos(n\Omega)=T_n\left[\cos(\Omega)\right] \qquad (4)$$

where $T_n[x]$ is the n-th degree Chebyshev polynomial, thus $H(e^{j\Omega})$ can be written as

$$H(e^{j\Omega})=\sum_{n=0}^{N}h(n)T_n[\cos(\Omega)] \qquad (5)$$

With the frequency response in this form we can now make a transformation of variables. For example, if we make the substitution

$$\cos(\Omega)=F(w).$$

We obtain another 1-D frequency response.

$$H(e^{jw})=\sum_{n=0}^{N}h(n)T_n[F(w)] \qquad (6)$$

We call $F(w)$ as subfilter(SF) and $H(e^{j\Omega})$ as prototype filter(PF). Another 1-D frequency response results if a 1-D transformation function is substituted for $\cos(\Omega)$.

Consider the specifications of the desired 1-D FIR filter as follows:

$$H(e^{jw})=\begin{cases}1\pm\delta_p, & for\ 0\le w\le w_p\\ 0\pm\delta_s, & for\ w_s\le w\le\pi\end{cases}. \qquad (7)$$

where w_p :passband edge, δ_p :passband deviation.

w_s :stopband edge, δ_s :stopband deviation.

To meet the desired specifications, the PF and SF must satisfy the following conditions:

(a)For the prototype filter,

Passband edge:

$$\Omega_p=\cos^{-1}(F(w_p)) \qquad (8a)$$

Stopband edge:

$$\Omega_s=\cos^{-1}(F(w_s)) \qquad (8b)$$

The deviations of passband and stopband are the same as that of the desiredl filter.

(b)For the subfilter,

$$\cos(\Omega_p)\le|F(w)|\le 1 \qquad (9a)$$

$$0\le|F(w)|\le\cos(\Omega_s) \qquad (9b)$$

From eqn.(7-9), the overall filter can be determined by the prototype filter and subfilter through the frequency transformation. The coefficients of the prototype filter and subfilter can be designed by the standard FIR design program. (

III. PROPOSED BANDPASS FUNCTION

For a bandpass FIR filter design by the frequency transformation method with a prescribed SF (PSF), the PSF can be a bandpass function. In this paper, the following new function is proposed.

$$PSF(w,w_0,k)=2(1-(\frac{\cos(w)-\cos(w_0)}{1+abs(\cos(w_0))})^2)^k-1, \qquad (10a)$$

where w_0 is a real value, k is an integer and $|w_0| \le \pi$. The function abs(x) is the absolute value of x. For the convenience, we define $q = (\frac{1}{1+abs(\cos(w_0))})^2$. The eqn.(10) can be rewritten as:

$$PSF(w, w_0, k) = 2(1 - q(\cos(w) - \cos(w_0))^2)^k - 1 \qquad (10b)$$

For k=1,

$$PSF(w, w_0, 1) = 1 - 2 * q(\cos(w) - \cos(w_0))^2 \qquad (10c)$$

The $PSF(w, w_0, k)$ function provide the rough shape of a bandpass filter. The maxmum value of $PSF(w, w_0, k)$ occurs at the point $w = w_0$. If a different value of w_0 is chosen, a different bandpass function determined by the function $PSF(w, w_0, k)$ is derived. The frequency response of $PSF(w, w_0, k)$ with the different values of w_0 is depicted in the figure 1.

As considering the implementation of the subfilter, the $PSF(w, w_0, k)$ function is rewritten as

$$PSF(w, w_0, k) = 2(A(w, w_0))^k - 1 \qquad (10d)$$

where $A(w, w_0) = 1 - q(\cos(w) - \cos(w_0))^2$

The structures to implement the $PSF(w, w_0, k)$ and $A(z)$ are shown in figure 2.

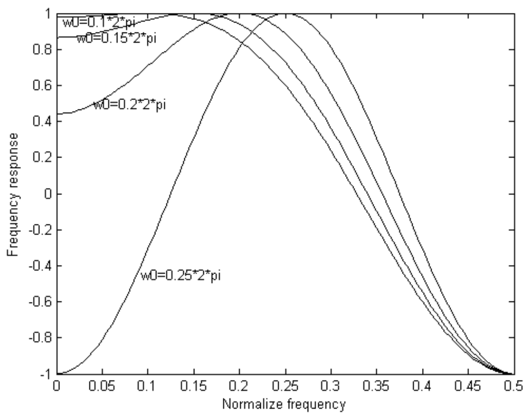

Figure 1. The plot of the frequency response of the function $BPF(w, w_0, 1)$ with the different values of w_0.

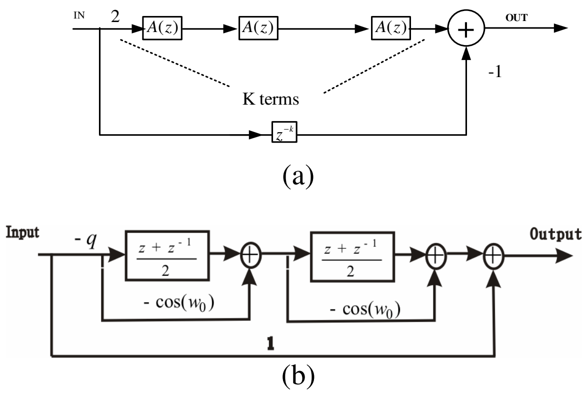

Figure 2. (a)The structure to impement the $PSF(w, w_0, k)$ function. (b)The structure to implement the $A(z)$ function.

By taking the replace,

$$F(w) = PSF(w, w_0, k)\,, \qquad (11)$$

in the eqn.(6), the resulting overall filter can be expressed as:

$$H(e^{jw}) = \sum_{n=0}^{N} h(n) T_n [PSF(w, w_0, k)] \qquad (12)$$

From the eqn. (8,9,12), it is easy to find that a bandpass filter can be derived by the transformation between the low-pass prototype filter and the prescribed subfilter. The number of multipliers required to implement the subfilter is 3k+1. If the value q and cos(w_0) are expressed as a sum of powers of two(SOPOT), to implement the subfilter requires no multipliers. To implemen the overall filter, the eqn.(12), requires N multipliers and (4k+2)N+1 adders. The multiplication rate is also N. For k=1, it requires N multipliers and 4N+1 adders. The overall filter length is 4kN+1.

The structure to implement the overall filter or eqn.(12) is proposed in the paper [4].

IV. FILTER DESIGN

Consider a desired bandpass filter with the following specifications:

$$H(e^{jw}) = \begin{cases} 0 \pm \delta_s & for\ 0 \le w \le w_{s1} \\ 1 \pm \delta_p & for\ w_{p1} \le w \le w_{p2} \\ 0 \pm \delta_s & for\ w_{s2} \le w \le \pi \end{cases} \qquad (13)$$

where w_{p1}, w_{p2} : passband edge frequency,

w_{s1}, w_{s2} : stopband edge frequency,

δ_p:passband deviation,

δ_s:stopband deviation.

Thus the PF and SF must satisfy some conditions simultaneously such that the overall filter meets the desired specifications. For the SF, $PSF(w, w_0, k)$, we must select a suitable value w_0 to satisfy the desired filter specifications. The PF, $H_p(e^{j\Omega})$, must satisfy the following conditions:

(a)The PF is a lowpass filter. The passband edge frequency of the PF:

$$\Omega_p = \max\,[\cos^{-1}(PSF(w_{p1}, w_0, k), \cos^{-1}(PSF(w_{p2}, w_0, k)] \tag{14a}$$

where the function max[a,b] is the maximum value of a and b. The stopband edge frequency of the prototype filter:

$$\Omega_S = \min\,[\cos^{-1}(PSF(w_{S1}, w_0, k), \cos^{-1}(PSF(w_{S2}, w_0, k)] \tag{14b}$$

where the function min[a,b] is the minimum value of a and b. From the eqn.(11,12,14), a bandpass filter design is derived.

Due to $PSF(w, w_0, k)$ being a powers of cos(w) with order k, it is easy to find that the function $PSF(w, w_0, k)$ possesses the sharper frequency response than that of the function cos(w). From the paper [8], an estimate of L of a 1-D FIR filter order can be found as:

$$L = \frac{-10\log_{10}\delta s\delta p - 13}{14.6\Delta f} \tag{15}$$

where Δf is the transition band width. It results that the order of the PF designed by the proposed method or the eqn.(12) is less than that directly designed by the standard FIR design program. If we carefully select the value of w_0, the order of the PF is reduced.

Example 1. Consider a desired bandpass filter with the following specifications:

$w_{p1}=(0.38)*\pi$, $w_{p2}=(0.42)*\pi$

$w_{s1}=(0.35)\pi$, $w_{s2}=(0.45)\pi$,

$\delta_p=0.01$, $\delta_s=0.01$.

The minimum order of a direct-form design [9] to meet these criteria is 140. It requirs 71 multipliers and 140 adders. The log magnitude response of the filter designed by the direct-form minimax method is shown in figure 3.

As designed by the proposed method, the w_0 can be slect as $w_0=0.4*\pi$. The value k=1 and $q = (\frac{1}{1+abs(\cos(0.2*2*\pi)})^2$ =0.5836. If q is approximated as a SOPOT form, q= 9/16. The resulting prototype filter specifications are shown as follows:

$\Omega_p=(0.0282)\pi$, $\Omega_s=(0.073)\pi$,

$\delta_p=0.01$, $\delta_s=0.01$.

The order of the prototype filter $H_p(e^{j\Omega})$ is 93, requiring 47 multipliers. The log magnitude responses of the resulting prototype filter and the overall filter are shown in figure 4. The overall filte requires 47 subfilter, $2(1-q(\cos(w)-\cos(w_0))^2)-1$, to satisfy the desired specifications. The number of multipliers required to implement the overall filter is 47. It is 66.2% of that designed by the direct form minimax method. It requires 189 adders to implement the overall filter. The overall filter length is 189. It is only 65%of that directly designed by the method[3].

The hardware requirements of the overall filter designed with the different $PSF(w, w_0, k)$ are shown in Table I. From Table I, we know that the order of the prototype filter is reduced as k is increased. The number of multipliers required to implement the overall filter is also futher reduced.

Table I

The hardware requirements of the overall filter designed with the proposed method with the different $PSF(w, w_0, k)$

method	k	q	NM	NA	O
Direct –form minimax method			71	140	140
proposed method	1	9/16	47	189	188
proposed method	2	5/16	43	431	344

NM: the number of multipliers required to implement the overall filter.

NA: the number of adders required to implement the overall filter.

O: the order of the overall filter.

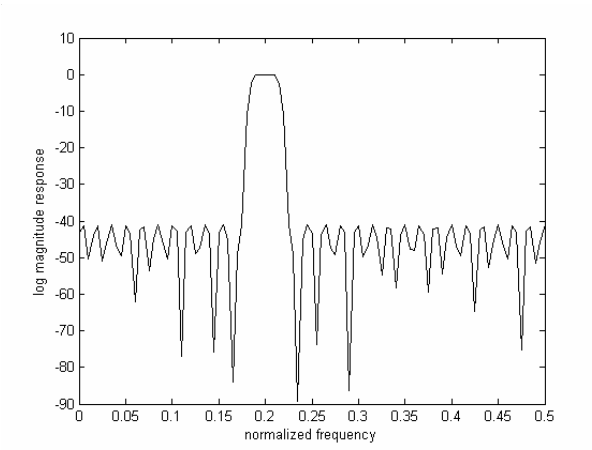

Figure 3. The plot of the log magnitude response for the filter designed by the direct form minimax method.

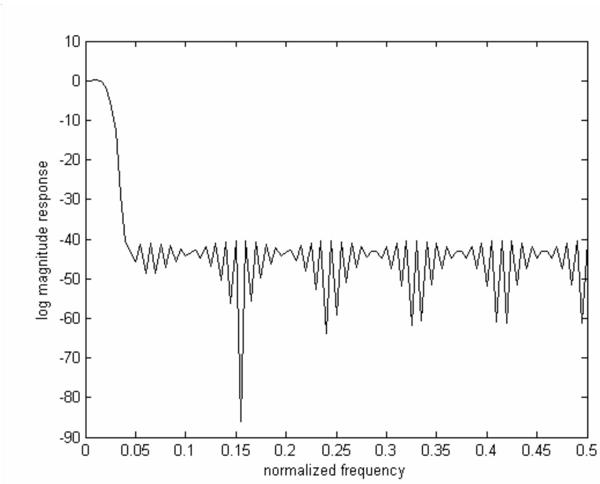

Figure 4(a). The plot of the log magnitude response for the prototype filter designed by the proposed method

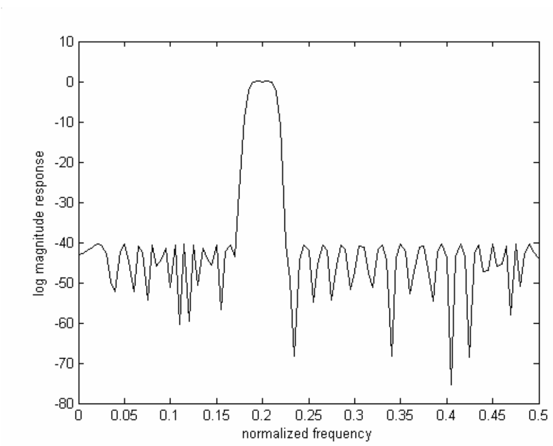

Figure 4(b). The plot of the log magnitude response for the overfilter filter designed by the proposed method

By combining the proposed method and the IFIR method proposed in [10], the design of a bandpass filter can be derived. The number of multipliers required to implement the overall filter can be further reduced to be very small.

V. CONCLUSIONS

In this paper, a frequency transformation method for the design of a bandpass filter is proposed. The $PSF(w, w_0, k)$ is proposed as the prescribed subfilter. To implement the function $PSF(w, w_0, k)$ requires no multipliers. It results that the number of multipliers and the multiplication rate of the overall filter is reduced. If the value k is increased, the number of multipliers to implement the overall filter is further reduced. It is also less than that designed by the direct form minimax method. The proposed method is efficient for the design of a bandpass filter. The method for the combination of the proposed method and the IFIR method will be developed in the future paper.

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