A New Method for Handling Fuzzy Multiple Attributes Group Decision-Making Problems

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Abstract— **In this paper, we present a new method to handle fuzzy multiple attributes group decision-making problems. First, we construct fuzzy importance matrices for decision-makers with respect to attributes and construct fuzzy evaluating matrices for decision-makers with respect to attributes of the alternatives. Then, based on the fuzzy importance matrices and the fuzzy evaluating matrices, we can get fuzzy rating matrices for decision-makers with respect to the alternatives. Then, we defuzzify the trapezoidal fuzzy numbers in the fuzzy rating matrices to get the rating matrices for the decision-makers. Then, we construct fuzzy preference matrices for the decision-makers with respect to the alternatives. Then, we calculate the average rating of each decision-maker with respect to the alternatives. Then, we sort these average ratings in a descending sequence and assign them different scores. Then, we calculate the summation values of the scores of the alternatives with respect to each decision-maker, respectively. The larger the summation values of the scores, the better the choice of the alternative. The proposed method is simpler than the methods presented in [2] and [11]. It provides us with a useful way to handle fuzzy multiple attributes group decision-making problems.**

Keywords— **Fuzzy Multiple Attributes Group Decision-Making, Linguistic Variables, Preference Matrix, Trapezoidal Fuzzy Numbers.**

1. INTRODUCTION

Multiple attributes decision making (MADM) is a process to select better alternatives from a set of alternatives by using a set of attributes. It can assist decision makers to make their decisions. In [8], Hwang and Yoon presented the techniques for the TOPSIS (The Order Preference by Similarity to Ideal Solution) method for handling the MADM problems. In the TOPSIS method, the best alternative has the shortest distance from the positive ideal solution and the farthest from the negative ideal solution, respectively. In recent years, some authors used the fuzzy set theory [21] to handle fuzzy decision making problems [1], [6], [12], [13] and fuzzy multiple attributes group decision-making problems [3], [5], [7], [9], [14], [16], [17], [19]. Fuzzy multiple attributes group decision-making is a flexible and useful method to handle decision-making problems. In [2], Chen presented a method to extend the TOPSIS method for group decision-making under a fuzzy environment. In [11], Li presented a compromise ratio methodology (CRM) for fuzzy multiple attributes group decision-making. In [18], Wang and Lee presented a method to generalizing TOPSIS for fuzzy multiple-criteria group decision-making. In [15], Rao et al. presented a novel combinatorial algorithm for the problems of fuzzy grey multi-attribute group decision-making. In [20], Wu and Chen presented the maximizing deviation method for group multiple attributes decision-making under a linguistic environment.

In this paper, we present a new method to handle fuzzy multiple attributes group decision-making problems. First, we construct fuzzy importance matrices for decision-makers with respect to attributes and construct fuzzy evaluating matrices for decision-makers with respect to attributes of the alternatives. Then, based on the fuzzy importance matrices and the fuzzy evaluating matrices, we can get fuzzy rating matrices for decision-makers with respect to alternatives. Then, we defuzzify the trapezoidal fuzzy numbers in the fuzzy rating matrices to get the rating matrices for the decision-makers. Then, we construct fuzzy preference matrices for the decision-makers with respect to the alternatives. Then, we calculate the average rating of each decision-maker with respect to the alternatives. Then, we sort these average ratings in a descending sequence and assign them different scores. Then, we calculate the summation values of the scores of the alternatives with respect to each decision-maker, respectively. The larger the summation value of the scores, the better the choice of the alternative. The proposed method is simpler than the methods presented in [2] and [11]. It provides us with a useful way to handle the fuzzy multiple attributes group decision-making problems.

2. ARITHMETIC OPERATIONS AND DEFUZZIFING OPERATIONS OF TRAPEZOIDAL FUZZY NUMBERS

In this section, we briefly review some arithmetic operations between trapezoidal fuzzy numbers form [10] and briefly review the defuzzifing operations of trapezoidal fuzzy numbers from [4]. A fuzzy number is a fuzzy set [21] which is both convex and normal. Fig. 1 shows a trapezoidal fuzzy number $\tilde{w}_i^k = (\alpha_i^k, \beta_i^k, \gamma_i^k, \delta_i^k)$, where $0 \le \alpha_i^k \le \beta_i^k \le \gamma_i^k \le \delta_i^k$.

***Definition 2.1* [10]:** Let $\tilde{z} = (z_1, z_2, z_3, z_4)$ and $\tilde{h} = (h_1, h_2, h_3, h_4)$ be two trapezoidal fuzzy numbers, where $0 \le z_1 \le z_2 \le z_3 \le z_4$ and $0 \le h_1 \le h_2 \le h_3 \le h_4$. The addition operation between $\tilde{z}$ and $\tilde{h}$ is defined by

$$\tilde{z} \oplus \tilde{h} = (z_1 + h_1, z_2 + h_2, z_3 + h_3, z_4 + h_4). \tag{1}$$

***Definition 2.2* [10]:** Let $\tilde{z} = (z_1, z_2, z_3, z_4)$ and $\tilde{h} = (h_1, h_2, h_3, h_4)$ be two trapezoidal fuzzy numbers, where $0 \le z_1 \le z_2 \le z_3 \le z_4$ and $0 \le h_1 \le h_2 \le h_3 \le h_4$. The multiplication operation between $\tilde{z}$ and $\tilde{h}$ is defined by

$$\tilde{z} \otimes \tilde{h} = (z_1 \times h_1, z_2 \times h_2, z_3 \times h_3, z_4 \times h_4). \tag{2}$$

***Definition 2.3* [10]:** Let $\tilde{z} = (z_1, z_2, z_3, z_4)$ and $\tilde{h} = (h_1, h_2, h_3, h_4)$ be two trapezoidal fuzzy numbers, where $0 \le z_1 \le z_2 \le z_3 \le z_4$ and $0 \le h_1 \le h_2 \le h_3 \le h_4$. The division operation between $\tilde{z}$ and $\tilde{h}$ is defined by

$$\tilde{z} \oslash \tilde{h} = \left(\frac{z_1}{h_4}, \frac{z_2}{h_3}, \frac{z_3}{h_2}, \frac{z_4}{h_1}\right). \tag{3}$$

***Definition 2.4* [4]:** Let $\tilde{z} = (z_1, z_2, z_3, z_4)$ be a trapezoidal fuzzy number, where $0 \le z_1 \le z_2 \le z_3 \le z_4$. The defuzzified value of the trapezoidal fuzzy number $\tilde{z}$ is equal to

$$\frac{z_1 + z_2 + z_3 + z_4}{4}. \tag{4}$$

3. A NEW METHOD FOR FUZZY MULTIPLE ATTRIBUTES GROUP DECISION-MAKING

In the section, we present a new method to handle fuzzy multiple attributes group decision-making problems. Assume that there are n alternatives x_1, x_2, $\cdots$, *and* x_n and assume that each alternative has m attributes f_1, f_2, $\cdots$, and f_m. Assume that there are p decision-makers D_1, D_2, $\cdots$, and D_p. Assume that the weight of the attribute f_i by the decision-maker D_k is represented by a trapezoidal fuzzy number $\tilde{w}_i^k$, where $\tilde{w}_i^k = (\alpha_i^k, \beta_i^k, \gamma_i^k, \delta_i^k)$, $1 \le i \le m$ and $1 \le k \le p$, as shown in Fig. 1.

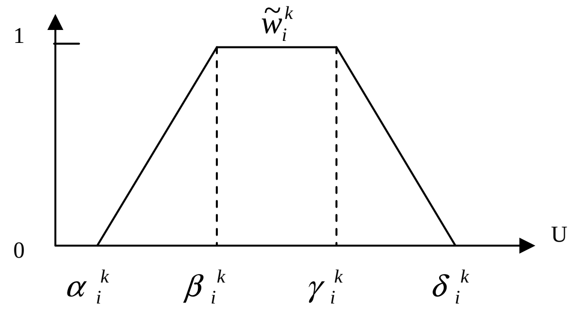

Fig. 1. Trapezoidal fuzzy number $\tilde{w}_i^k$

Assume that the fuzzy evaluating value of the attribute f_i of the alternative x_j given by the decision-maker D_k is represented by a trapezoidal fuzzy number $\tilde{f}_{ij}^k$, where $\tilde{f}_{ij}^k = (a_{ij}^k, b_{ij}^k, c_{ij}^k, d_{ij}^k)$, $1 \le i \le m$, $1 \le j \le n$ and $1 \le k \le p$, as shown in Fig. 2.

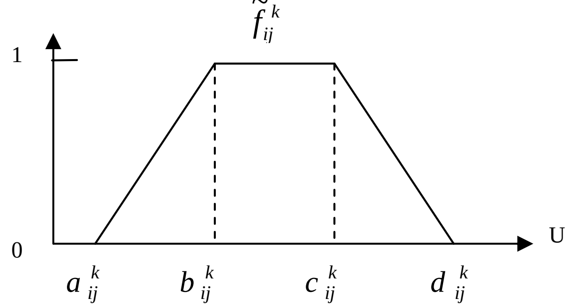

Fig. 2. Trapezoidal fuzzy number $\tilde{f}_{ij}^k$

Construct the fuzzy importance matrix $\tilde{W}_k$ for decision-maker D_k with respect to attributes, shown as follows:

$$\tilde{W}_k = (\tilde{w}_i^k)_{1\times m} = \begin{array}{c} \begin{array}{cccc} f_1 & f_2 & \cdots & f_m \end{array} \\ \begin{bmatrix} \tilde{w}_1^k & \tilde{w}_2^k & \cdots & \tilde{w}_m^k \end{bmatrix} \end{array}, \quad (5)$$

where $1 \le i \le m$ and $1 \le k \le p$.

Construct the fuzzy evaluating matrix $\tilde{Y}_k$ for decision-maker D_k with respect to attribute f_i of the alternative x_j, shown as follows:

$$\tilde{Y}_k = (\tilde{f}_{ij}^k)_{m\times n} = \begin{array}{c} \\ f_1 \\ f_2 \\ \vdots \\ f_m \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} \tilde{f}_{11}^k & \tilde{f}_{12}^k & \cdots & \tilde{f}_{1n}^k \\ \tilde{f}_{21}^k & \tilde{f}_{22}^k & \cdots & \tilde{f}_{2n}^k \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{f}_{m1}^k & \tilde{f}_{m2}^k & \cdots & \tilde{f}_{mn}^k \end{bmatrix} \end{array}, \quad (6)$$

where $1 \le i \le m$, $1 \le j \le n$ and $1 \le k \le p$.

Step 1: Based on the fuzzy importance matrix $\tilde{W}_k$ and the fuzzy evaluating matrix $\tilde{Y}_k$, we can get fuzzy rating matrix $\tilde{Z}_k$ for decision-maker D_k with respect to alternative,

$$\tilde{Z}_k = (\tilde{F}_j^k)_{1\times n} = \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} \tilde{F}_1^k & \tilde{F}_2^k & \cdots & \tilde{F}_n^k \end{bmatrix} \end{array},$$

$$\tilde{F}_j^k = \left((\tilde{w}_1^k \otimes \tilde{f}_{1j}^k) \oplus (\tilde{w}_2^k \otimes \tilde{f}_{2j}^k) \oplus (\tilde{w}_3^k \otimes \tilde{f}_{3j}^k) \oplus \cdots \oplus (\tilde{w}_m^k \otimes \tilde{f}_{mj}^k)\right) \oslash \left(\tilde{w}_1^k \oplus \tilde{w}_2^k \oplus \tilde{w}_3^k \oplus \cdots \oplus \tilde{w}_m^k\right), \quad (7)$$

where $1 \le j \le n$ and $1 \le k \le p$, $\tilde{F}_j^k$ is a trapezoidal fuzzy number denoting the fuzzy rating value for decision-maker D_k with respect to alternative x_j, $\tilde{F}_j^k = (\mu_j^k, \eta_j^k, \rho_j^k, \lambda_j^k)$, $\tilde{w}_i^k$ denotes the weight of the attribute f_i given by the decision-maker D_k, $\tilde{f}_{ij}^k$ denotes the fuzzy evaluating value of the attribute f_i of alternative x_j given by the decision-maker D_k, where $1 \le i \le m$, $1 \le j \le n$ and $1 \le k \le p$.

Step 2: Based on Eq. (2), defuzzified each fuzzy number $\tilde{F}_j^k$ in the fuzzy rating matrix into F_j^k to construct the rating matrix Z_k for the decision-maker D_k, shown as follows:

$$Z_k = (F_j^k)_{1\times n} = \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} F_1^k & F_2^k & \cdots & F_n^k \end{bmatrix} \end{array},$$

where

$$F_j^k = \frac{(\mu_j^k + \eta_j^k + \rho_j^k + \lambda_j^k)}{4}, \quad (8)$$

$1 \le j \le n$ and $1 \le k \le p$.

Step 3: Construct the fuzzy preference matrix $T_k = (t_{gq}^k)_{n\times n}$ for the decision-maker D_k with respect to the alternative x_j, where $1 \le g \le n$, $1 \le q \le n$ and $1 \le k \le p$,

$$T_k = (t_{gq}^k)_{n\times n} = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & \cdots & x_n \end{array} \\ \begin{bmatrix} t_{11}^k & t_{12}^k & t_{13}^k & \cdots & t_{1n}^k \\ t_{21}^k & t_{22}^k & t_{23}^k & \cdots & t_{2n}^k \\ t_{31}^k & t_{32}^k & t_{33}^k & \cdots & t_{3n}^k \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ t_{n1}^k & t_{n2}^k & t_{n3}^k & \cdots & t_{nn}^k \end{bmatrix} \end{array},$$

$$t_{gq}^k = \frac{F_g^k}{F_g^k + F_q^k}, \quad (9)$$

where t_{gq}^k denotes the preference rating from x_g to x_q.

Step 4: Based on the preference matrix T_k obtained in Step 3, calculate the average rating value r_j^k for decision-maker D_k with respect to alternative x_j,

$$r_j^k = \frac{\sum_{q=1}^{n} t_{jq}^k}{n}, \quad (10)$$

where $1 \le j \le n$ and $1 \le k \le p$.

Step 5: Sort the average rating values r_1^k, r_2^k, $\cdots$, and r_n^k for each k in a descending sequence, where $1 \le k \le p$. If the maximum average rating value of the sorting sequence is r_j^k, then let v_j^k is assigned *n* scores with respect to the alternative x_j (i.e., let $v_j^k = n$), where $1 \le j \le n$ and $1 \le k \le p$; if the second maximum average rating value of the sorting sequence is r_j^k, then v_j^k is assigned *n-1* scores with respect to the alternative x_j (i.e., let $v_j^k = n$-1) where $1 \le j \le n$ and $1 \le k \le p$; …; if the minimum average rating value of the sorting sequence is r_j^k, then v_j^k is assigned one score with respect to the alternative x_j (i.e., let v_j^k = 1) ,where $1 \le j \le n$ and $1 \le k \le p$. Assume that the ranking score for the decision-maker D_k with respect to the alternative x_j is v_j^k, where $1 \le j \le n$ and $1 \le k \le p$. Calculate the summation value V_j of the scores of the alternative x_j with respect to the decision-makers D_1, D_2, $\cdots$, and D_p, respectively, shown as follows:

$$V_j = \sum_{k=1}^{p} v_j^k, \tag{11}$$

where $1 \le j \le n$. The larger the value of V_j, the better the choice of the alternative x_j, where $1 \le j \le n$.

4. NUMERICAL EXAMPLE

In this section, we use an example to illustrate the proposed method to handle fuzzy multiple attributes group decision-making problems.

Example 4.1 [11]: Assume that a company wants to develop a new food product and assume that there are three decision-makers D_1, D_2 and D_3 who want to choose the best food product. Assume that there are three alternative food products x_1, x_2 and x_3 and five attributes, i.e. colourful (denoted by f_1), taste (denoted by f_2), smell (denoted by f_3), profit (denoted by f_4) and expiration Date (denoted by f_5). Assume that the three decision-makers use linguistic variables "Very Low" (VL), "Low" (L), "Medium Low" (ML), "Medium" (M), "Medium High" (MH), "High" (H) and "Very High" (VH) as shown in Table 1 to describe the importance of each attribute. Assume that the importance of the attributes given by the decision-makers is shown in Table 2. Assume that decision-makers use the linguistic variables "Very Poor" (VP), "Poor" (P), "Medium Poor" (MP), "Fair" (F), "Medium Good" (MG), "Good" (G) and "Very Good" (VG) shown in Table 3 for the ratings of each alternative with respect to each attribute. The fuzzy evaluating values of the alternatives given by the decision-makers with respect to different attributes are shown in Table 4. Based on Table 1, Table 2 and Eq. (5), we can construct the fuzzy importance matrices $\tilde{W}_1$, $\tilde{W}_2$, and $\tilde{W}_3$, for the decision-makers D_1, D_2 and D_3, respectively, shown as follows:

$$\tilde{W}_1 = \begin{array}{c} \begin{array}{ccccc} f_1 & f_2 & f_3 & f_4 & f_5 \end{array} \\ \begin{bmatrix} \tilde{w}_1^1 & \tilde{w}_2^1 & \tilde{w}_3^1 & \tilde{w}_4^1 & \tilde{w}_5^1 \end{bmatrix} \end{array},$$

$$\tilde{W}_2 = \begin{array}{c} \begin{array}{ccccc} f_1 & f_2 & f_3 & f_4 & f_5 \end{array} \\ \begin{bmatrix} \tilde{w}_1^2 & \tilde{w}_2^2 & \tilde{w}_3^2 & \tilde{w}_4^2 & \tilde{w}_5^2 \end{bmatrix} \end{array},$$

$$\tilde{W}_3 = \begin{array}{c} \begin{array}{ccccc} f_1 & f_2 & f_3 & f_4 & f_5 \end{array} \\ \begin{bmatrix} \tilde{w}_1^3 & \tilde{w}_2^3 & \tilde{w}_3^3 & \tilde{w}_4^3 & \tilde{w}_5^3 \end{bmatrix} \end{array},$$

where

$\tilde{w}_1^1 = (0.7, 0.9, 0.9, 1.0)$, $\tilde{w}_2^1 = (0.9, 1.0, 1.0, 1.0)$, $\tilde{w}_3^1 = (0.9, 1.0, 1.0, 1.0)$, $\tilde{w}_4^1 = (0.9, 1.0, 1.0, 1.0)$, $\tilde{w}_5^1 = (0.3, 0.5, 0.5, 0.7)$, $\tilde{w}_1^2 = (0.9, 1.0, 1.0, 1.0)$, $\tilde{w}_2^2 = (0.9, 1.0, 1.0, 1.0)$, $\tilde{w}_3^2 = (0.7, 0.9, 0.9, 1.0)$, $\tilde{w}_4^2 = (0.9, 1.0, 1.0, 1.0)$, $\tilde{w}_5^2 = (0.5, 0.7, 0.7, 0.9)$, $\tilde{w}_1^3 = (0.5, 0.7, 0.7, 0.9)$, $\tilde{w}_2^3 = (0.9, 1.0, 1.0, 1.0)$, $\tilde{w}_3^3 = (0.7, 0.9, 0.9, 1.0)$, $\tilde{w}_4^3 = (0.9, 1.0, 1.0, 1.0)$, $\tilde{w}_5^3 = (0.5, 0.7, 0.7, 0.9)$.

TABLE 1
LINGUISTIC VARIABLES FOR REPRESENTING THE IMPORTANCE OF EACH ATTRIBUTE AND THEIR CORRESPONDING TRAPEZOIDAL FUZZY NUMBERS [11]

Linguistic Variables	Trapezoidal Fuzzy Numbers
Very Low (VL)	(0, 0, 0, 0.1)
Low (L)	(0, 0.1, 0.1, 0.3)
Medium Low (ML)	(0.1, 0.3, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.5, 0.7)
Medium High (MH)	(0.5, 0.7, 0 .7, 0.9)
High (H)	(0.7, 0.9, 0.9, 1)
Very High (VH)	(0.9, 1, 1, 1)

TABLE 2
THE IMPORTANCE OF THE ATTRIBUTES GIVEN BY THE DECISION-MAKERS [11]

Attributes	Decision-Makers		
	D_1	D_2	D_3
f_1	H	VH	MH
f_2	VH	VH	VH
f_3	VH	H	H
f_4	VH	VH	VH
f_5	M	MH	MH

Based on Table 3, Table 4 and Eq. (6), construct the fuzzy evaluating matrices $\tilde{Y}_1$, $\tilde{Y}_2$ and $\tilde{Y}_3$ for the decision-makers D_1, D_2 and D_3, respectively, shown as follows:

$$\tilde{Y}_1 = \begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} \tilde{f}_{11}^1 & \tilde{f}_{12}^1 & \tilde{f}_{13}^1 \\ \tilde{f}_{21}^1 & \tilde{f}_{22}^1 & \tilde{f}_{23}^1 \\ \tilde{f}_{31}^1 & \tilde{f}_{32}^1 & \tilde{f}_{33}^1 \\ \tilde{f}_{41}^1 & \tilde{f}_{42}^1 & \tilde{f}_{43}^1 \\ \tilde{f}_{51}^1 & \tilde{f}_{52}^1 & \tilde{f}_{53}^1 \end{bmatrix} \end{array},$$

$$\tilde{Y}_2 = \begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} \tilde{f}_{11}^{\,2} & \tilde{f}_{12}^{\,2} & \tilde{f}_{13}^{\,2} \\ \tilde{f}_{21}^{\,2} & \tilde{f}_{22}^{\,2} & \tilde{f}_{23}^{\,2} \\ \tilde{f}_{31}^{\,2} & \tilde{f}_{32}^{\,2} & \tilde{f}_{33}^{\,2} \\ \tilde{f}_{41}^{\,2} & \tilde{f}_{42}^{\,2} & \tilde{f}_{43}^{\,2} \\ \tilde{f}_{51}^{\,2} & \tilde{f}_{52}^{\,2} & \tilde{f}_{53}^{\,2} \end{bmatrix} \end{array},$$

$$\tilde{Y}_3 = \begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} \tilde{f}_{11}^{\,3} & \tilde{f}_{12}^{\,3} & \tilde{f}_{13}^{\,3} \\ \tilde{f}_{21}^{\,3} & \tilde{f}_{22}^{\,3} & \tilde{f}_{23}^{\,3} \\ \tilde{f}_{31}^{\,3} & \tilde{f}_{32}^{\,3} & \tilde{f}_{33}^{\,3} \\ \tilde{f}_{41}^{\,3} & \tilde{f}_{42}^{\,3} & \tilde{f}_{43}^{\,3} \\ \tilde{f}_{51}^{\,3} & \tilde{f}_{52}^{\,3} & \tilde{f}_{53}^{\,3} \end{bmatrix} \end{array},$$

where

$\tilde{f}_{11}^1$ = (5, 7, 7, 9), $\tilde{f}_{12}^1$ = (7, 9, 9, 10),
$\tilde{f}_{13}^1$ = (9, 10, 10, 10), $\tilde{f}_{21}^1$ = (7, 9, 9, 10),
$\tilde{f}_{22}^1$ = (9, 10, 10, 10), $\tilde{f}_{23}^1$ = (5, 7, 7, 9),
$\tilde{f}_{31}^1$ = (3, 5, 5, 7), $\tilde{f}_{32}^1$ = (9, 10, 10, 10),
$\tilde{f}_{33}^1$ = (7, 9, 9, 10), $\tilde{f}_{41}^1$ = (9, 10, 10, 10),
$\tilde{f}_{42}^1$ = (9, 10, 10, 10), $\tilde{f}_{43}^1$ = (7, 9, 9, 10),
$\tilde{f}_{51}^1$ =(3, 5, 5, 7), $\tilde{f}_{52}^1$ =(9, 10, 10, 10),
$\tilde{f}_{53}^1$ =(7, 9, 9, 10), $\tilde{f}_{11}^2$ = (7, 9, 9, 10),
$\tilde{f}_{12}^2$ = (7, 9, 9, 10), $\tilde{f}_{13}^2$ = (7, 9, 9, 10),
$\tilde{f}_{21}^2$ = (5, 7, 7, 9), $\tilde{f}_{22}^2$ = (9, 10, 10, 10),
$\tilde{f}_{23}^2$ = (7, 9, 9, 10), $\tilde{f}_{31}^2$ = (7, 9, 9, 10),
$\tilde{f}_{32}^2$ = (9, 10, 10, 10), $\tilde{f}_{33}^2$ = (5, 7, 7, 9),
$\tilde{f}_{41}^2$ = (7, 9, 9, 10), $\tilde{f}_{42}^2$ = (9, 10, 10, 10),
$\tilde{f}_{43}^2$ = (9, 10, 10, 10), $\tilde{f}_{51}^2$ =(3, 5, 5, 7),
$\tilde{f}_{52}^2$ =(5, 7, 7, 9), $\tilde{f}_{53}^2$ =(7, 9, 9, 10),
$\tilde{f}_{11}^3$ = (5, 7, 7, 9), $\tilde{f}_{12}^3$ = (5, 7, 7, 9),
$\tilde{f}_{13}^3$ = (3, 5, 5, 7), $\tilde{f}_{21}^3$ = (3, 5, 5, 7),
$\tilde{f}_{22}^3$ = (9, 10, 10, 10), $\tilde{f}_{23}^3$ = (9, 10, 10, 10),
$\tilde{f}_{31}^3$ = (7, 9, 9, 10), $\tilde{f}_{32}^3$ = (7, 9, 9, 10),
$\tilde{f}_{33}^3$ = (9, 10, 10, 10), $\tilde{f}_{41}^3$ = (9, 10, 10, 10),
$\tilde{f}_{42}^3$ = (9, 10, 10, 10), $\tilde{f}_{43}^3$ = (5, 7, 7, 9),
$\tilde{f}_{51}^3$ =(3, 5, 5, 7), $\tilde{f}_{52}^3$ =(7, 9, 9, 10),
$\tilde{f}_{53}^3$ =(5, 7, 7, 9),

[Step 1] Based on Eqs. (1)–(3) and Eq. (7), construct the fuzzy rating matrices $\tilde{Z}_1$, $\tilde{Z}_2$ and $\tilde{Z}_3$ for the decision-makers D_1, D_2 and D_3, respectively, shown as follows：

$$\tilde{Z}_1 = \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} \tilde{F}_1^1 & \tilde{F}_2^1 & \tilde{F}_3^1 \end{bmatrix} \end{array},$$

$$\tilde{Z}_2 = \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} \tilde{F}_1^2 & \tilde{F}_2^2 & \tilde{F}_3^2 \end{bmatrix} \end{array},$$

$$\tilde{Z}_3 = \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} \tilde{F}_1^3 & \tilde{F}_2^3 & \tilde{F}_3^3 \end{bmatrix} \end{array},$$

where

$\tilde{F}_1^1 = \left((\tilde{w}_1^1 \otimes \tilde{f}_{11}^1) \oplus (\tilde{w}_2^1 \otimes \tilde{f}_{21}^1) \oplus (\tilde{w}_3^1 \otimes \tilde{f}_{31}^1) \oplus (\tilde{w}_4^1 \otimes \tilde{f}_{41}^1) \oplus (\tilde{w}_5^1 \otimes \tilde{f}_{51}^1)\right)$
$\oslash \left(\tilde{w}_1^1 \oplus \tilde{w}_2^1 \oplus \tilde{w}_3^1 \oplus \tilde{w}_4^1 \oplus \tilde{w}_5^1\right)$
= (((0.7, 0.9, 0.9, 1.0) $\otimes$ (5, 7, 7, 9)) $\oplus$ ((0.9, 1.0, 1.0, 1.0) $\otimes$ (7, 9, 9, 10)) $\oplus$ ((0.9, 1.0, 1.0, 1.0) $\otimes$ (3, 5, 5, 7)) $\oplus$ ((0.9, 1.0, 1.0, 1.0) $\otimes$ (9, 10, 10, 10)) $\oplus$ ((0.3, 0.5, 0.5, 0.7) $\otimes$ (7, 9, 9, 10))) $\oslash$ ((0.7, 0.9, 0.9, 1.0) $\oplus$ (0.9, 1.0, 1.0, 1.0) $\oplus$ (0.9, 1.0, 1.0, 1.0) $\oplus$ (0.9, 1.0, 1.0, 1.0) $\oplus$ (0.3, 0.5, 0.5, 0.7))
= (4.57, 7.54, 7.54, 11.05),

$\tilde{F}_2^1 = \left((\tilde{w}_1^1 \otimes \tilde{f}_{12}^1) \oplus (\tilde{w}_2^1 \otimes \tilde{f}_{22}^1) \oplus (\tilde{w}_3^1 \otimes \tilde{f}_{32}^1) \oplus (\tilde{w}_4^1 \otimes \tilde{f}_{42}^1) \oplus (\tilde{w}_5^1 \otimes \tilde{f}_{52}^1)\right)$
$\oslash \left(\tilde{w}_1^1 \oplus \tilde{w}_2^1 \oplus \tilde{w}_3^1 \oplus \tilde{w}_4^1 \oplus \tilde{w}_5^1\right)$
= (((0.7, 0.9, 0.9, 1.0) $\otimes$ (7, 9, 9, 10)) $\oplus$ ((0.9, 1.0, 1.0, 1.0) $\otimes$ (9, 10, 10, 10)) $\oplus$ ((0.9, 1.0, 1.0, 1.0) $\otimes$ (9, 10, 10, 10)) $\oplus$ ((0.9, 1.0, 1.0, 1.0) $\otimes$ (9, 10, 10, 10)) $\oplus$ ((0.3, 0.5, 0.5, 0.7) $\otimes$ (9, 10, 10, 10))) $\oslash$ ((0.7, 0.9, 0.9, 1.0) $\oplus$ (0.9, 1.0, 1.0, 1.0) $\oplus$ (0.9, 1.0, 1.0, 1.0) $\oplus$ (0.9, 1.0, 1.0, 1.0) $\oplus$ (0.3, 0.5, 0.5, 0.7))
= (6.79, 9.80, 9.80, 12.70),

$\tilde{F}_3^1 = \left((\tilde{w}_1^1 \otimes \tilde{f}_{13}^1) \oplus (\tilde{w}_2^1 \otimes \tilde{f}_{23}^1) \oplus (\tilde{w}_3^1 \otimes \tilde{f}_{33}^1) \oplus (\tilde{w}_4^1 \otimes \tilde{f}_{43}^1) \oplus (\tilde{w}_5^1 \otimes \tilde{f}_{53}^1)\right)$
$\oslash \left(\tilde{w}_1^1 \oplus \tilde{w}_2^1 \oplus \tilde{w}_3^1 \oplus \tilde{w}_4^1 \oplus \tilde{w}_5^1\right)$
= (((0.7, 0.9, 0.9, 1.0) $\otimes$ (9, 10, 10, 10)) $\oplus$ ((0.9, 1.0, 1.0, 1.0) $\otimes$ (5, 7, 7, 9)) $\oplus$ ((0.9, 1.0, 1.0, 1.0) $\otimes$ (7, 9, 9, 10)) $\oplus$ ((0.9, 1.0, 1.0, 1.0) $\otimes$ (7, 9, 9, 10)) $\oplus$ ((0.3, 0.5, 0.5, 0.7) $\otimes$ (7, 9, 9, 10))) $\oslash$ ((0.7, 0.9, 0.9, 1.0) $\oplus$ (0.9, 1.0, 1.0, 1.0) $\oplus$ (0.9, 1.0, 1.0, 1.0) $\oplus$ (0.9, 1.0, 1.0, 1.0) $\oplus$ (0.3, 0.5, 0.5, 0.7))
= (5.43, 8.75, 8.75, 12.43),

$\tilde{F}_1^2 = \left((\tilde{w}_1^2 \otimes \tilde{f}_{11}^2) \oplus (\tilde{w}_2^2 \otimes \tilde{f}_{21}^2) \oplus (\tilde{w}_3^2 \otimes \tilde{f}_{31}^2) \oplus (\tilde{w}_4^2 \otimes \tilde{f}_{41}^2) \oplus (\tilde{w}_5^2 \otimes \tilde{f}_{51}^2)\right)$
$\oslash \left(\tilde{w}_1^2 \oplus \tilde{w}_2^2 \oplus \tilde{w}_3^2 \oplus \tilde{w}_4^2 \oplus \tilde{w}_5^2\right)$
= (((0.9, 1.0, 1.0, 1.0) $\otimes$ (7, 9, 9, 10)) $\oplus$ ((0.9, 1.0, 1.0, 1.0) $\otimes$ (5, 7, 7, 9)) $\oplus$ ((0.7, 0.9, 0.9, 1.0) $\otimes$ (7, 9, 9, 10)) $\oplus$ ((0.9, 1.0, 1.0, 1.0) $\otimes$ (7, 9, 9, 10)) $\oplus$ ((0.5, 0.7, 0.7, 0.9) $\otimes$ (3, 5, 5, 7))) $\oslash$ ((0.9, 1.0, 1.0, 1.0) $\oplus$ (0.9, 1.0, 1.0, 1.0) $\oplus$

(0.7, 0.9, 0.9, 1.0) ⊕ (0.9, 1.0, 1.0, 1.0) ⊕ (0.5, 0.7, 0.7, 0.9))
= (4.80, 7.96, 7.96, 11.62),

$$\tilde{F}_2^2 = \left((\tilde{w}_1^2 \otimes \tilde{f}_{12}^2) \oplus (\tilde{w}_2^2 \otimes \tilde{f}_{22}^2) \oplus (\tilde{w}_3^2 \otimes \tilde{f}_{32}^2) \oplus (\tilde{w}_4^2 \otimes \tilde{f}_{42}^2) \oplus (\tilde{w}_5^2 \otimes \tilde{f}_{52}^2)\right) \varnothing \left(\tilde{w}_1^2 \oplus \tilde{w}_2^2 \oplus \tilde{w}_3^2 \oplus \tilde{w}_4^2 \oplus \tilde{w}_5^2\right)$$

= (((0.9, 1.0, 1.0, 1.0) ⊗ (7, 9, 9, 10)) ⊕ ((0.9, 1.0, 1.0, 1.0) ⊗ (9, 10, 10, 10)) ⊕ ((0.7, 0.9, 0.9, 1.0) ⊗ (9, 10, 10, 10)) ⊕ ((0.9, 1.0, 1.0, 1.0) ⊗ (9, 10, 10, 10)) ⊕ ((0.5, 0.7, 0.7, 0.9) ⊗ (5, 7, 7, 9))) ∅ ((0.9, 1.0, 1.0, 1.0) ⊕ (0.9, 1.0, 1.0, 1.0) ⊕ (0.7, 0.9, 0.9, 1.0) ⊕ (0.9, 1.0, 1.0, 1.0) ⊕ (0.5, 0.7, 0.7, 0.9))
= (6.39, 9.33, 9.33, 12.33),

$$\tilde{F}_3^2 = \left((\tilde{w}_1^2 \otimes \tilde{f}_{13}^2) \oplus (\tilde{w}_2^2 \otimes \tilde{f}_{23}^2) \oplus (\tilde{w}_3^2 \otimes \tilde{f}_{33}^2) \oplus (\tilde{w}_4^2 \otimes \tilde{f}_{43}^2) \oplus (\tilde{w}_5^2 \otimes \tilde{f}_{53}^2)\right) \varnothing \left(\tilde{w}_1^2 \oplus \tilde{w}_2^2 \oplus \tilde{w}_3^2 \oplus \tilde{w}_4^2 \oplus \tilde{w}_5^2\right)$$

= (((0.9, 1.0, 1.0, 1.0) ⊗ (7, 9, 9, 10)) ⊕ ((0.9, 1.0, 1.0, 1.0) ⊗ (7, 9, 9, 10)) ⊕ ((0.7, 0.9, 0.9, 1.0) ⊗ (5, 7, 7, 9)) ⊕ ((0.9, 1.0, 1.0, 1.0) ⊗ (9, 10, 10, 10)) ⊕ ((0.5, 0.7, 0.7, 0.9) ⊗ (7, 9, 9, 10))) ∅ ((0.9, 1.0, 1.0, 1.0) ⊕ (0.9, 1.0, 1.0, 1.0) ⊕ (0.7, 0.9, 0.9, 1.0) ⊕ (0.9, 1.0, 1.0, 1.0) ⊕ (0.5, 0.7, 0.7, 0.9))
= (5.65, 8.83, 8.83, 12.31),

$$\tilde{F}_1^3 = \left((\tilde{w}_1^3 \otimes \tilde{f}_{11}^3) \oplus (\tilde{w}_2^3 \otimes \tilde{f}_{21}^3) \oplus (\tilde{w}_3^3 \otimes \tilde{f}_{31}^3) \oplus (\tilde{w}_4^3 \otimes \tilde{f}_{41}^3) \oplus (\tilde{w}_5^3 \otimes \tilde{f}_{51}^3)\right) \varnothing \left(\tilde{w}_1^3 \oplus \tilde{w}_2^3 \oplus \tilde{w}_3^3 \oplus \tilde{w}_4^3 \oplus \tilde{w}_5^3\right)$$

= (((0.5, 0.7, 0.7, 0.9) ⊗ (5, 7, 7, 9)) ⊕ ((0.9, 1.0, 1.0, 1.0) ⊗ (3, 5, 5, 7)) ⊕ ((0.7, 0.9, 0.9, 1.0) ⊗ (7, 9, 9, 10)) ⊕ ((0.9, 1.0, 1.0, 1.0) ⊗ (9, 10, 10, 10)) ⊕ ((0.5, 0.7, 0.7, 0.9) ⊗ (3, 5, 5, 7))) ∅ ((0.5, 0.7, 0.7, 0.9) ⊕ (0.9, 1.0, 1.0, 1.0) ⊕ (0.7, 0.9, 0.9, 1.0) ⊕ (0.9, 1.0, 1.0, 1.0) ⊕ (0.5, 0.7, 0.7, 0.9))
= (4.10, 7.33, 7.33, 11.83),

$$\tilde{F}_2^3 = \left((\tilde{w}_1^3 \otimes \tilde{f}_{12}^3) \oplus (\tilde{w}_2^3 \otimes \tilde{f}_{22}^3) \oplus (\tilde{w}_3^3 \otimes \tilde{f}_{32}^3) \oplus (\tilde{w}_4^3 \otimes \tilde{f}_{42}^3) \oplus (\tilde{w}_5^3 \otimes \tilde{f}_{52}^3)\right) \varnothing \left(\tilde{w}_1^3 \oplus \tilde{w}_2^3 \oplus \tilde{w}_3^3 \oplus \tilde{w}_4^3 \oplus \tilde{w}_5^3\right)$$

= (((0.5, 0.7, 0.7, 0.9) ⊗ (5, 7, 7, 9)) ⊕ ((0.9, 1.0, 1.0, 1.0) ⊗ (9, 10, 10, 10)) ⊕ ((0.7, 0.9, 0.9, 1.0) ⊗ (7, 9, 9, 10)) ⊕ ((0.9, 1.0, 1.0, 1.0) ⊗ (9, 10, 10, 10)) ⊕ ((0.5, 0.7, 0.7, 0.9) ⊗ (7, 9, 9, 10))) ∅ ((0.5, 0.7, 0.7, 0.9) ⊕ (0.9, 1.0, 1.0, 1.0) ⊕ (0.7, 0.9, 0.9, 1.0) ⊕ (0.9, 1.0, 1.0, 1.0) ⊕ (0.5, 0.7, 0.7, 0.9))
= (7.74, 9.14, 9.14, 9.8),

$$\tilde{F}_3^3 = \left((\tilde{w}_1^3 \otimes \tilde{f}_{13}^3) \oplus (\tilde{w}_2^3 \otimes \tilde{f}_{23}^3) \oplus (\tilde{w}_3^3 \otimes \tilde{f}_{33}^3) \oplus (\tilde{w}_4^3 \otimes \tilde{f}_{43}^3) \oplus (\tilde{w}_5^3 \otimes \tilde{f}_{53}^3)\right) \varnothing \left(\tilde{w}_1^3 \oplus \tilde{w}_2^3 \oplus \tilde{w}_3^3 \oplus \tilde{w}_4^3 \oplus \tilde{w}_5^3\right)$$

= (((0.5, 0.7, 0.7, 0.9) ⊗ (3, 5, 5, 7)) ⊕ ((0.9, 1.0, 1.0, 1.0) ⊗ (9, 10, 10, 10)) ⊕ ((0.7, 0.9, 0.9, 1.0) ⊗ (9, 10, 10, 10)) ⊕ ((0.9, 1.0, 1.0, 1.0) ⊗ (5, 7, 7, 9)) ⊕ ((0.5, 0.7, 0.7, 0.9) ⊗ (5, 7, 7, 9))) ∅ ((0.5, 0.7, 0.7, 0.9) ⊕ (0.9, 1.0, 1.0, 1.0) ⊕ (0.7, 0.9, 0.9, 1.0) ⊕ (0.9, 1.0, 1.0, 1.0) ⊕ (0.5, 0.7, 0.7, 0.9))
= (6.54, 8, 8, 9.04).

TABLE 3

LINGUISTIC VARIABLES FOR THE RATINGS OF EACH ALTERNATIVE WITH RESPECT TO EACH ATTRIBUTE AND THEIR CORRESPONDING TRAPEZOIDAL FUZZY NUMBERS [11]

Linguistic Variables	Trapezoidal Fuzzy Numbers
Very poor (VP)	(0, 0, 0, 1)
Poor (P)	(0, 1, 1, 3)
Medium poor (MP)	(1, 3, 3, 5)
Fair (F)	(3, 5, 5, 7)
Medium good (MG)	(5, 7, 7, 9)
Good (G)	(7, 9, 9, 1)
Very good (VG)	(9, 1, 1, 1)

[Step 2] Based on Eq. (4) and Eq. (8), construct the rating matrices Z_1, Z_2 and Z_3 for the decision-makers D_1, D_2 and D_3, respectively, shown as follows:

$$Z_1 = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 \end{matrix} \\ \begin{bmatrix} F_1^1 & F_2^1 & F_3^1 \end{bmatrix} \end{array},$$

$$Z_2 = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 \end{matrix} \\ \begin{bmatrix} F_1^2 & F_2^2 & F_3^2 \end{bmatrix} \end{array},$$

$$Z_3 = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 \end{matrix} \\ \begin{bmatrix} F_1^3 & F_2^3 & F_3^3 \end{bmatrix} \end{array},$$

where

$$F_1^1 = \frac{4.57 + 7.54 + 7.54 + 11.05}{4} = 7.68,$$

$$F_2^1 = \frac{6.79 + 9.8 + 9.8 + 12.7}{4} = 9.77,$$

$$F_3^1 = \frac{5.43 + 8.75 + 8.75 + 12.43}{4} = 8.84,$$

$$F_1^2 = \frac{4.8 + 7.96 + 7.96 + 11.62}{4} = 8.1,$$

$$F_2^2 = \frac{6.39 + 9.33 + 9.33 + 12.33}{4} = 9.34,$$

$F_3^2 = \dfrac{5.65+8.83+8.83+12.31}{4} = 8.9,$

$F_1^3 = \dfrac{4.1+7.33+7.33+11.83}{4} = 7.65,$

$F_2^3 = \dfrac{7.74+9.14+9.14+9.8}{4} = 8.96,$

$F_3^3 = \dfrac{6.54+8+8+9.04}{4} = 7.9.$

TABLE 4
EVALUATING VALUES OF THE ALTERNATIVES GIVEN BY THE DECISION-MAKERS WITH RESPECT TO DIFFERENT ATTRIBUTES [11]

Attributes	Alternatives	Decision-Makers		
		D_1	D_2	D_3
f_1	x_1	MG	G	MG
	x_2	G	G	MG
	x_3	VG	G	F
f_2	x_1	G	MG	F
	x_2	VG	VG	VG
	x_3	MG	G	VG
f_3	x_1	F	G	G
	x_2	VG	VG	G
	x_3	G	MG	VG
f_4	x_1	VG	G	VG
	x_2	VG	VG	VG
	x_3	G	VG	MG
f_5	x_1	F	F	F
	x_2	VG	MG	G
	x_3	G	G	MG

[Step 3] Based on Eq. (9), construct the fuzzy preference matrices T_1, T_2 and T_3 for the decision-makers D_1, D_2 and D_3, respectively, shown as follows：

$$T_1 = \begin{array}{c c} & \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} t_{11}^1 & t_{12}^1 & t_{13}^1 \\ t_{21}^1 & t_{22}^1 & t_{23}^1 \\ t_{31}^1 & t_{32}^1 & t_{33}^1 \end{bmatrix} \end{array},$$

$$T_2 = \begin{array}{c c} & \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} t_{11}^2 & t_{12}^2 & t_{13}^2 \\ t_{21}^2 & t_{22}^2 & t_{23}^2 \\ t_{31}^2 & t_{32}^2 & t_{33}^2 \end{bmatrix} \end{array},$$

$$T_3 = \begin{array}{c c} & \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} t_{11}^3 & t_{12}^3 & t_{13}^3 \\ t_{21}^3 & t_{22}^3 & t_{23}^3 \\ t_{31}^3 & t_{32}^3 & t_{33}^3 \end{bmatrix} \end{array},$$

where

$t_{11}^1 = \dfrac{F_1^1}{F_1^1+F_1^1} = \dfrac{7.68}{7.68+7.68} = 0.5,$

$t_{12}^1 = \dfrac{F_1^1}{F_1^1+F_2^1} = \dfrac{7.68}{7.68+9.77} = 0.44,$

$t_{13}^1 = \dfrac{F_1^1}{F_1^1+F_3^1} = \dfrac{7.68}{7.68+8.84} = 0.46,$

$t_{21}^1 = \dfrac{F_2^1}{F_2^1+F_1^1} = \dfrac{9.77}{9.77+7.68} = 0.56,$

$t_{22}^1 = \dfrac{F_2^1}{F_2^1+F_2^1} = \dfrac{9.77}{9.77+9.77} = 0.5,$

$t_{23}^1 = \dfrac{F_1^1}{F_1^1+F_1^1} = \dfrac{9.77}{9.77+8.84} = 0.52,$

$t_{31}^1 = \dfrac{F_3^1}{F_3^1+F_1^1} = \dfrac{8.84}{8.84+7.68} = 0.54,$

$t_{32}^1 = \dfrac{F_3^1}{F_3^1+F_2^1} = \dfrac{8.84}{8.84+9.77} = 0.48,$

$t_{33}^1 = \dfrac{F_3^1}{F_3^1+F_3^1} = \dfrac{8.84}{8.84+8.84} = 0.5,$

$t_{11}^2 = \dfrac{F_1^2}{F_1^2+F_1^2} = \dfrac{8.1}{8.1+8.1} = 0.5,$

$t_{12}^2 = \dfrac{F_1^2}{F_1^2+F_2^2} = \dfrac{8.1}{8.1+9.34} = 0.46,$

$t_{13}^2 = \dfrac{F_1^2}{F_1^2+F_3^2} = \dfrac{8.1}{8.1+8.9} = 0.48,$

$t_{21}^2 = \dfrac{F_2^2}{F_2^2+F_1^2} = \dfrac{9.34}{9.34+8.1} = 0.54,$

$t_{22}^2 = \dfrac{F_2^2}{F_2^2+F_2^2} = \dfrac{9.34}{9.34+9.34} = 0.5,$

$t_{23}^2 = \dfrac{F_2^2}{F_2^2+F_3^2} = \dfrac{9.34}{9.34+8.9} = 0.51,$

$t_{31}^2 = \dfrac{F_3^2}{F_3^2+F_1^2} = \dfrac{8.9}{8.9+8.1} = 0.52,$

$t_{32}^2 = \dfrac{F_3^2}{F_3^2+F_2^2} = \dfrac{8.9}{8.9+9.34} = 0.49,$

$t_{33}^2 = \dfrac{F_3^2}{F_3^2+F_3^2} = \dfrac{8.9}{8.9+8.9} = 0.5,$

$t_{11}^3 = \dfrac{F_1^3}{F_1^3+F_1^3} = \dfrac{7.65}{7.65+7.65} = 0.5,$

$$t_{12}^3 = \frac{F_1^3}{F_1^3 + F_2^3} = \frac{7.65}{7.65 + 8.96} = 0.46,$$

$$t_{13}^3 = \frac{F_1^3}{F_1^3 + F_3^3} = \frac{7.65}{7.65 + 7.9} = 0.49,$$

$$t_{21}^3 = \frac{F_2^3}{F_2^3 + F_1^3} = \frac{8.96}{8.96 + 7.65} = 0.54,$$

$$t_{22}^3 = \frac{F_2^3}{F_2^3 + F_2^3} = \frac{8.96}{8.96 + 8.96} = 0.5,$$

$$t_{23}^3 = \frac{F_2^3}{F_2^3 + F_3^3} = \frac{8.96}{8.96 + 7.9} = 0.53,$$

$$t_{31}^3 = \frac{F_3^3}{F_3^3 + F_1^3} = \frac{7.9}{7.9 + 7.65} = 0.51,$$

$$t_{32}^3 = \frac{F_3^3}{F_3^3 + F_2^3} = \frac{7.9}{7.9 + 8.96} = 0.47,$$

$$t_{33}^3 = \frac{F_3^3}{F_3^3 + F_3^3} = \frac{7.9}{7.9 + 7.9} = 0.5.$$

[Step 4] Based on Eq. (10), calculate the average rating r_j^k for the decision-makers D_k with respect to alternative x_j, where $1 \leq j \leq 3$ and $1 \leq k \leq 3$, shown as follows:

$$r_1^1 = \frac{t_{11}^1 + t_{12}^1 + t_{13}^1}{3} = \frac{0.5 + 0.44 + 0.46}{3} = 0.47,$$

$$r_2^1 = \frac{t_{21}^1 + t_{22}^1 + t_{23}^1}{3} = \frac{0.56 + 0.5 + 0.52}{3} = 0.53,$$

$$r_3^1 = \frac{t_{31}^1 + t_{32}^1 + t_{33}^1}{3} = \frac{0.54 + 0.48 + 0.5}{3} = 0.51,$$

$$r_1^2 = \frac{t_{11}^2 + t_{12}^2 + t_{13}^2}{3} = \frac{0.5 + 0.46 + 0.48}{3} = 0.48,$$

$$r_2^2 = \frac{t_{21}^2 + t_{22}^2 + t_{23}^2}{3} = \frac{0.54 + 0.5 + 0.51}{3} = 0.52,$$

$$r_3^2 = \frac{t_{31}^2 + t_{32}^2 + t_{33}^2}{3} = \frac{0.52 + 0.49 + 0.5}{3} = 0.5,$$

$$r_1^3 = \frac{t_{11}^3 + t_{12}^3 + t_{13}^3}{3} = \frac{0.5 + 0.46 + 0.49}{3} = 0.48,$$

$$r_2^3 = \frac{t_{21}^3 + t_{22}^3 + t_{23}^3}{3} = \frac{0.54 + 0.5 + 0.53}{3} = 0.52,$$

$$r_3^3 = \frac{t_{31}^3 + t_{32}^3 + t_{33}^3}{3} = \frac{0.51 + 0.47 + 0.5}{3} = 0.49.$$

[Step 5] Sort the average rating values r_1^k, r_2^k, and r_3^k for each k in a descending sequence, where $1 \leq k \leq 3$, shown as follows:

$r_2^1 = 0.53 > r_3^1 = 0.51 > r_1^1 = 0.47$,

$r_2^2 = 0.48 > r_3^2 = 0.5 > r_1^2 = 0.48$,

$r_2^3 = 0.52 > r_3^3 = 0.49 > r_1^3 = 0.48$.

Because $r_2^1 > r_3^1 > r_1^1$, we can see that the scores are $v_2^1 = 3$, $v_3^1 = 2$ and $v_1^1 = 1$. Because $r_2^2 > r_3^2 > r_1^2$, we can see that the scores are $v_2^2 = 3$, $v_3^2 = 2$ and $v_1^2 = 1$. Because $r_2^3 > r_3^3 > r_1^3$, we can see that the scores are $v_2^3 = 3$, $v_3^3 = 2$ and $v_1^3 = 1$. Based on Eq. (11), calculate the summation values V_1, V_2 and V_3 of the scores, respectively, shown as follows:

$V_1 = v_1^1 + v_1^2 + v_1^3 = 1+1+1 = 3,$

$V_2 = v_2^1 + v_2^2 + v_2^3 = 3+3+3 = 9,$

$V_3 = v_3^1 + v_3^2 + v_3^3 = 2+2+2 = 6.$

Because $V_2 > V_3 > V_1$, we can see that the alternative x_2 is the best choice among x_1, x_2 and x_3. This result coincides with the result shown in [2] and [11].

5. CONCLUSIONS

In this paper, we have presented a new method to handle fuzzy multiple attributes group decision-making problems. First, we construct fuzzy importance matrices for decision-makers with respect to attributes and construct fuzzy evaluating matrices for decision-makers with respect to the attributes of the alternatives. Then, based on the fuzzy importance matrices and the fuzzy evaluating matrices, we can get fuzzy rating matrices for the decision-makers with respect to the alternatives. Then, we defuzzify the trapezoidal fuzzy numbers in the fuzzy rating matrices to get the rating matrices for the decision-makers. Then, we construct fuzzy preference matrices for the decision-makers with respect to the alternatives. Then, we calculate the average rating of each decision-maker with respect to the alternatives. Then, we sort these average ratings in a descending sequence and assign them different scores. Then, we calculate the summation values of the scores of the alternatives with respect to each decision-maker, respectively. The larger the summation value of the scores, the better the choice of the alternative. The proposed method is simpler than the methods presented in [2] and [11]. It provides us with a useful way to handle fuzzy multiple attributes group decision-making problems.

ACKNOWLEDGEMENTS

This work was supported in part by the National Science Council, Republic of China, under Grant NSC 97-2221-E-011-108.

REFERENCES

[1] R. E. Bellman and L. A. Zadeh, "Decision-making in a fuzzy environment," *Management Sciences*, vol. 17, no. 4, pp. 141–164, 1970.

[2] C. T. Chen, "Extensions of the TOPSIS for group decision-making under fuzzy environment," *Fuzzy Sets and Systems*, vol. 114, no. 1, pp. 1–9, 2000.

[3] F. Chiclana, E. Herrera-Viedma, F. Herrera, and S. Alonso, "Some induced ordered weighted averaging operators and their use for solving group decision-making problems based on fuzzy preference relations," *European Journal of Operational Research*, vol. 182, no. 1, pp. 383–399, 2007.

[4] S. M. Chen, "Evaluating weapon systems using fuzzy arithmetic operations," *Fuzzy sets and systems*, vol. 77, no. 3, pp. 265–276, 1996.

[5] S. M. Chen, "fuzzy group decision making for evaluating the rate of aggregative risk in software development", *Fuzzy Sets and Systems*, vol. 118, no. 1, pp. 75–88, 2001

[6] M. Delgado, J. L. Verdegay, and M. A. Vila, "Linguistic decision-making models," *International Journal of Intelligent Systems*, vol. 7, no. 5, pp. 479–492, 1992.

[7] Z. P. Fan, G. F. Hu, and S. H. Xiao, "A method for multiple attribute decision-making with the fuzzy preference relation on alternatives," *Computers & Industrial Engineering*, vol. 46, no. 2, pp. 321–327, 2004.

[8] C. L. Hwang and K. Yoon, *Multiple Attributes Decision Making Methods and Applications*, Springer, Berlin Heidelberg, 1981.

[9] F. Herrera, E. Herrera-Viedma, and J. L. Verdegay, "A model of consensus in group decision making under linguistic assessments," *Fuzzy Sets and Systems*, vol. 78, no. 1, pp. 73–87, 1996.

[10] A. Kaufman and M. M. Gupta, *Introduction to Fuzzy Arithmetic Theory and Applications*, New York: Van Nostrand Reinhold, 1991.

[11] D. F. Li, "Compromise ratio method for fuzzy multi-attribute group decision making," *Applied Soft Computing Journal*, vol. 7, no. 3, pp. 807-817, 2007.

[12] D. F. Li, "Multiattribute decision making models and methods using intuitionistic fuzzy sets," *Journal of Computer and System Sciences*, vol. 70, no. 1, pp. 73–85, 2005.

[13] D. F. Li, "An approach to fuzzy multiattribute decision making under uncertainty," *Information Sciences*, vol. 169, no. 1–2, pp. 97–112, 2005.

[14] H. S. Lee, "Optimal consensus of fuzzy opinions under group decision making environment," *Fuzzy Sets and Systems*, vol. 132, no. 3, pp. 303–315, 2002.

[15] C. J. Rao, X. P. Xiao, and J. Peng, "Novel combinatorial algorithm for the problems of fuzzy grey multi-attribute group decision making," *Journal of Systems Engineering and Electronics*, vol. 18, no. 4, pp. 774-780, 2007.

[16] J. Wang and Y. I. Lin, "A fuzzy multicriteria group decision making approach to select configuration items for software development," *Fuzzy Sets and Systems*, vol. 134, no. 3, pp. 343–363, 2003.

[17] R. C. Wang and S. J. Chuu, "Group decision-making using a fuzzy linguistic approach for evaluating the flexibility in a manufacturing system," *European Journal of Operational Research*, vol. 154, no. 3, pp. 563–572, 2004.

[18] Y. J. Wang and H. S. Lee, "Generalizing TOPSIS for fuzzy multiple-criteria group decision-making," *Computers and Mathematics with Applications*, vol. 53, no. 11, pp. 1762–1772, 2007.

[19] Y. M. Wang and C. Parkan, "Two new approaches for assessing the weights of fuzzy opinions in group decision analysis," *Information Sciences*, vol. 176, no. 23, pp. 3538–3555, 2006.

[20] Z. B. Wu and Y. H. Chen, "The maximizing deviation method for group multiple attribute decision making under linguistic environment," *Fuzzy Sets and Systems*, vol. 158, no. 14, pp. 1608–1617, 2007.

[21] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.