Proxy Agent for Combinatorial Reverse Auction with Multiple Buyers

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***Abstract*—Although combinatorial reverse auction has attracted much attention recently, most studies focus on problems with single buyer and multiple sellers. In this paper, we will study combinatorial reverse auction with multiple buyers and multiple sellers. We propose the concept of proxy buyer to deal with this problem. The proxy buyer consolidates the demands from the buyers and then holds a reverse auction to try to obtain the goods from a set of sellers who can provide the goods. Each seller places bids for each bundle of goods he can provide. The problem is to determine the winners to minimize the total cost to acquire the required items. The main results include: (1) a problem formulation for the combinatorial reverse auction problem; (2) a solution methodology based on Lagrangian relaxation and (3) analysis of numerical results based on our solution algorithms.**

***Keywords*—Combinatorial auction, e-commerce, optimization.**

1. INTRODUCTION

Auctions are popular, distributed and autonomy preserving ways of allocating items or tasks among multiple agents to maximize revenue or minimize cost. Applying combinatorial auctions in corporations' procurement processes can lead to significant savings [10] and [11]. Allowing bids for bundles of items is the foundation of combinatorial auctions, which have attracted considerable attention in the auction literature. There are, however, several problems with the implementation of combinatorial auctions. Combinatorial auctions have been notoriously difficult to solve from a computational point of view [12]. Combinatorial auction is closely related to the set packing/knapsack problem [13]. It deals with computational aspects and heuristics for solving what is known as the Winner Determination Problem of an auction [14], [15] and [16].

An excellent survey on combinatorial auctions can be found in [1] and [3]. In a combinatorial auction [1], bidders may place bids on combinations of items or tasks. This allows the bidders to express complementarities between items instead of having to speculate into an item's valuation about the impact of possibly getting other, complementary items or tasks. The combinatorial auction problem can be modeled as a set packing problem (SPP), a well-known NP-complete problem [4]-[8]. Many algorithms have been developed for combinatorial auction problems. For example, in [2], [17], [18], the authors proposed a Lagrangian Heuristic for a combinatorial auction problem. Exact algorithms have been developed for the SPP problem, including a branch and bound search [8], iterative deepening A* search [7] and the direct application of available CPLEX IP solver [4]. However, in real world, combinatorial reverse auction may take place with multiple buyers and multiple sellers. Motivated by the deficiency of the existing studies, we consider a combinatorial auction problem in which there are multiple buyers and multiple sellers. We propose the concept of proxy buyer to deal with this problem. The proxy buyer consolidates the demands from the buyers and then holds a reverse auction to try to obtain the goods from a set of sellers who can provide the goods. Each seller places bids for each bundle of goods he can provide. The problem is to determine the winners to minimize the total cost for the proxy buyer.

The remainder of this paper is organized as follows. In Section 2, we present the winner determination problem formulation for proxy buyer's combinatorial reverse auction problem. In Section 3, we propose the Lagrangian relaxation algorithms. In Section 4, specification of the requirements for the implementation of our solution algorithms is proposed and an economic interpretation for our solution approach is given. In Section 5, analysis of numerical results and the proposed algorithm are made. We conclude this paper in Section 6.

2. COMBINATORIAL REVERSE AUCTION WITH PROXY BUYER

In this paper, we first formulate the above combinatorial optimization problem as an integer programming problem. We then develop solution algorithms based on Lagrangian relaxation. Figure 1 illustrates an application scenario in which Buyer requests to purchase at least a bundle of items 2A, 4B and 3C from the market. There are four bidders, Seller 1, Seller 2, Seller 3 and Seller4 who place bids in the system. Suppose Seller 1 places one bid: (2A, 2B, p1), where p1 denote the prices of the bid. Seller 2 places one bid: (2A, 2C, p2). Seller 3 places one bid: (2B, 2C, p3). Seller 4 places one bid: (1C, p4). We assume that all the bids entered the auction are recorded. A bid is said to be active if it is in the solution. We assume that there is only one bid active for all the bids placed by the same bidder. For this example, the solution for this reverse auction problem is Seller1: (2A, 2B, p1), Seller 3: (2B, 2C, p3) and Seller 4: (1C, p4).

Consider a buyer who requests a set of items to be purchased. Let K denote the number of items requested. Let d_{ik} denote the desired units of the $k-th$ items by the buyer, where $k \in \{1,2,3,....,K\}$. In a combinatorial auction, there are many bidders to submit a tender. Let N denote the number of bidders in a combinatorial auction. Each $n \in \{1,2,3,....,N\}$ represents a bidder. To model the combinatorial reverse auction problem, the bid must be represented mathematically. We use a vector $b_n = (q_{n1}, q_{n2}, q_{n3}, ..., q_{nk}, p_n)$ to represent the bid submitted by bidder n, where q_{nk} is a nonnegative integer that denotes the quantity of the $k-th$ items and p_n is a real positive number that denotes the price of the bundle. As the quantity of the $k-th$ items cannot exceed the quantity d_{ik}, it follows that the constraint $0 \le q_{nk} \le d_{ik}$ must be satisfied. The bid b_n is actually an offer to deliver q_{nk} units of items for each $k \in \{1,2,3,....,K\}$ a total price of p_n. Let N denote the number of bids placed by bidder $n \in \{1,2,3,....,N\}$. To formulate the problem, we use the variable x_n to indicate the bid placed by bidder n is active (x_n =1) or inactive (x_n =0). The winner determination problem can be formulated as an Integer Programming problem as follows.

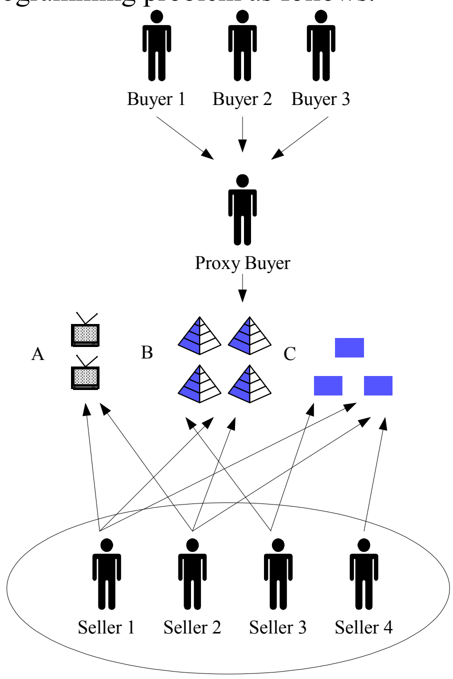

Figure 1 Combinatorial Reverse Auction with Proxy Buyer

Winner Determination Problem (WDP):

$$\min \sum_{n=1}^{N} x_n p_n$$

$$s.t. \sum_{n=1}^{N} x_n q_{nk} \ge \sum_{i=1}^{I} d_{ik} \; \forall k = 1,2,...,K \qquad (2-1)$$

$$x_n \in \{0,1\} \qquad (2-2)$$

In WDP problem, we observe that the coupling among different operations is caused by the contention for resources through the minimal resource requirement constraints (2-1).

3. SOLUTION ALGORITHM

For a given Lagrange multiplier λ, the relaxation of constraints (2-1) decomposes the original problem into a number of bidder's subproblems (BS). These subproblems can be solved independently. That is, the Lagrangian relaxation results in subproblems with a highly decentralized decision making structure. Interactions among subproblems are reflected through Lagrange multipliers, which are determined by solving the following dual problem.

$\max_{\lambda \geq 0} L(\lambda)$, where

$$L(\lambda) = \min \sum_{n=1}^{N} x_n p_n + \sum_{k=1}^{K} \lambda_k \Big(\sum_{i=1}^{I} d_{ik} - \big(\sum_{n=1}^{N} x_n q_{nk}\big)\Big)$$
$$s.t.\ x_n \in \{0,1\}$$
$$= \sum_{i=1}^{I} \sum_{k=1}^{K} \lambda_k d_{ik} + \min \sum_{n=1}^{N} x_n \Big(p_n - \sum_{k=1}^{K} \lambda_k q_{nk}\Big)$$
$$s.t.\ x_n \in \{0,1\}$$
$$= \sum_{i=1}^{I} \sum_{k=1}^{K} \lambda_k d_{ik} + \sum_{n=1}^{N} L_n(\lambda), \textit{ with}$$
$$L_n(\lambda) = \min\Big(p_n - \sum_{k=1}^{K} \lambda_k q_{nk}\Big)$$
$$s.t.\ x_n \in \{0,1\}$$

$L_n(\lambda)$ defines a bidder's subproblems (BS). Our methodology for finding a near optimal solution of WDP is developed based on the result of Lagrangian relaxation and decomposition. It consists of three parts as follows.

(1) An algorithm for solving subproblems

Given λ, the optimal solution to BS subproblem $L_n(\lambda)$ can be solved as follows.

$$x_n = \begin{cases} 1 \textit{ if } P_n - \sum_{k=1}^{K} \lambda_k q_{nk} < 0 \\ 0 \textit{ if } P_n - \sum_{k=1}^{K} \lambda_k q_{nk} \geq 0 \end{cases}$$

(2) A subgradient method for solving the dual problem $\max_{\lambda \geq 0} L(\lambda)$

Let x^l be the optimal solution to the subproblems for given Lagrange multipliers λ^l of iteration l. We define the subgradient of $L(\lambda)$ as

$$g_k^l = \left.\frac{\partial L(\lambda)}{\partial \lambda_k}\right|_{\lambda_k^l} = \sum_{i=1}^{I} d_{ik} - \Big(\sum_{n=1}^{N} x_n q_{nk}\Big) \quad ,$$

where $k \in \{1,2,...,K\}$.

The subgradient method proposed by Polyak [9] is adopted to update λ as follows

$$\lambda_k^{l+1} = \begin{cases} \lambda_k^l + \alpha^l g_k^l \textit{ if } \lambda_k^l + \alpha^l \lambda_k^l \geq 0; \\ 0 \textit{ otherwise.} \end{cases}$$

where $\alpha^l = c\dfrac{\overline{L} - L(\lambda)}{\Sigma_k (g_k^l)^2}$, $0 \leq c \leq 2$ and $\overline{L}$ is an estimate of the optimal dual cost. The iteration step terminates if α^l is smaller than a threshold. Polyak proved that this method has a linear convergence rate.

Iterative application of the algorithms in (1) and (2) may converge to an optimal dual solution (x^*, λ^*).

(3) A heuristic algorithm for finding a near-optimal $\bar{x}$, feasible solution based on the solution (x^*, λ^*) of the relaxed problem

The solution (x^*, λ^*) may result in one type of constraint violation due to relaxation: assignment of the quantity of items less than the demand of the items. Our heuristic scheme first checks all the demand constraints $\sum_{n=1}^{N} x_n q_{nk} \geq \sum_{i=1}^{I} d_{ik}\ \forall k = 1,2,...,K$ that have not been satisfied.

Let $K^0 = \{k | k \in \{1,2,3,....,K\}, \sum_{n=1}^{N} x_n q_{nk} < \sum_{i=1}^{I} d_{ik}\}$.

K^0 denotes the set of demand constraints violated. Let $N^0 = \{n | n \in \{1,2,3,....,N\}, x_n^* = 0\}$. N^0 denotes the set of bidders that is not a winner in solution x^* . To make the set of constraints K^0 satisfied, we first pick $k \in K^0$ with

$$k = \arg \min_{k \in K^0} \sum_{i=1}^{I} d_{ik} - \sum_{n=1}^{N} x_n^* q_{nk} .$$

The heuristic algorithm proceeds as follows to make constraint k satisfied. Select $n \in N^0$ with $n = arg \min_{n \in \{1,2,...,N\}, q_{nk} > 0} p_n$ and set x_n^* =1. After performing the above operation, we set $N^0 \leftarrow N^0 \setminus \{n\}$. If the violation of the k -th constraint cannot be completely resolved, the same procedure repeats. Eventually, all the constraints will be satisfied. We use $\bar{x}$ to denote the resulting feasible solution obtained from the above heuristics.

4. NUMERICAL RESULTS AND ANALYSIS

Based on the proposed algorithms for combinatorial reverse auction, we conduct several examples to illustrate the validity of our method.

Example 1: Consider two buyers who will purchase a set of items as specified in Table 1. Four potential sellers' bids as shown in Table 2. For this example, we have $I = 2, \quad N = 4, \quad K = 3, d_{11} = 1, d_{12} = 2, d_{13} = 1, d_{21} = 3,$ $d_{22} = 0, d_{23} = 1$. According to Table 4.2, we have:

$q_{11} = 2, q_{12} = 1, q_{13} = 1$, $q_{21} = 1, q_{22} = 2, q_{23} = 1$, $q_{31} = 3, q_{32} = 0, q_{33} = 2$, $q_{41} = 1, q_{42} = 1, q_{43} = 1$,

Suppose the prices of the bids are: $p_1 = 45, p_2 = 45, p_3 = 55, p_4 = 32,$

	Item 1	Item 2	Item 3
Buyer 1	1	2	1
Buyer 2	3	0	1

Table 1 Buyers' Requirements

	Item 1	Item 2	Item 3
Seller 1	2	1	1
Seller 2	1	2	1
Seller 3	3	0	2
Seller 4	1	1	1

Table 2 Sellers' Bids

Suppose we initialize the Lagrange multipliers as follows.

$\lambda(1) = 10.0, \lambda(2) = 15.0, \lambda(3) = 10.0$.

Our algorithm the subgradient algorithm converges to the following solution:

x_3^* =1and x_n^* =0 for all the other n . As the above solution is a feasible one, the heuristic algorithm needs not be applied. Therefore, $\bar{x}_3$ =1, $\bar{x}_2$ =1. The solution x^* is also an optimal solution. The duality gap of the solution is 3.75%. The duality gap is within 5%. This means the solution methodology generates near optimal solution.

Despite the duality gap is not zero, the solution $\bar{x}$ is also an optimal solution for this example.

Table 3 illustrates the duality gap of several cases based on the problem size (I, K). According the results, the duality gaps are within 3%. This means the solution methodology generates near optimal solution.

I	N	K	Duality Gap
5	70	5	2.65%
10	50	5	2.37%
10	30	24	1.15%

Table 3

In addition to the two examples above, we also conduct several experiments to study the computational efficiency of our proposed algorithms.

Figure 2 shows the CPU time for a number of problems in which parameter N and K are fixed while the parameter I is changed. The increase in the CPU time is not significant as parameter I is increased.

According to the following equation:

$$L(\lambda)=\sum_{i=1}^{I}\sum_{k=1}^{K}\lambda_k d_{ik}+\sum_{n=1}^{N}L_n(\lambda) \qquad ,$$

$$L_n(\lambda)$$

where $L_n(\lambda)=\min(p_n-\sum_{k=1}^{K}\lambda_k q_{nk})$

$$s.t.\ x_n\in\{0,1\}$$

This result justifies the fact that the CPU time to compute $L(\lambda)$ for a given λ grows approximately proportionally to I .

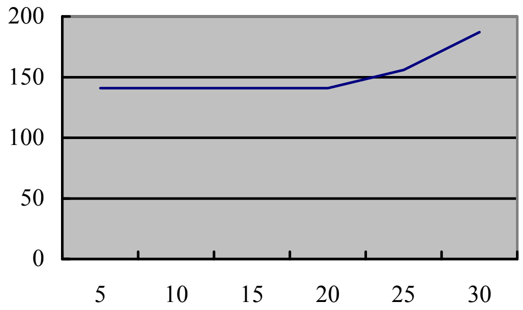

Figure 2 CPU time with respect to I

Figure 3 shows the CPU time for a number of problems in which parameter I and K are fixed while the parameter N is changed. The increase in the CPU time is not significant as parameter N is increased.

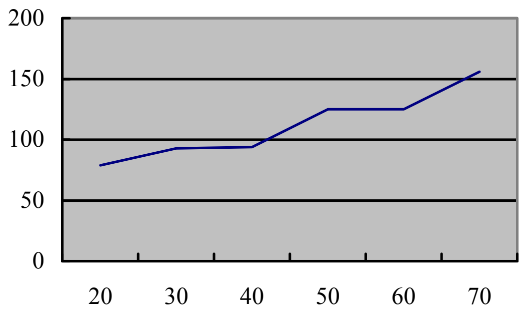

Figure 3 CPU time with respect to N

Figure 4 shows the CPU time for a number of problems in which parameter I and N are fixed while the parameter K is changed. The increase in the CPU time is not significant as parameter K is increased.

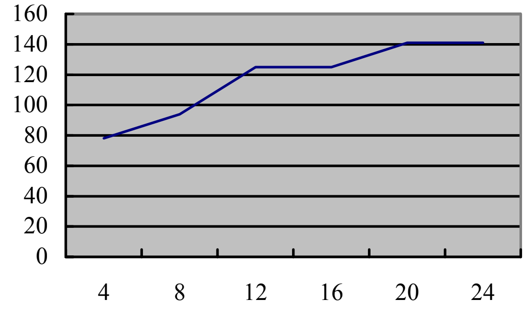

Figure 4 CPU time with respect to K

5. CONCLUSIONS

Most studies on combinatorial reverse auction focus on auction with single buyer/multiple sellers. A practical issue is how to handle combinatorial reverse auction with multiple buyers and multiple sellers. Combinatorial auction enables several bidders to bid on different combination of goods simultaneously according to personal preferences. Bidders can select multiple items at one time and offer those items a combined price. We propose the concept of proxy buyer to deal with this problem. The proxy buyer consolidates the demands from the buyers and then holds a reverse auction to try to obtain the goods from a set of sellers who can provide the goods. Each seller places bids for each bundle of goods he can provide. We formulate a winner determination optimization problem for combinatorial auction with a proxy buyer. The demands of the proxy buyer impose additional constraints on determination of the winners.The problem is to determine the winners to minimize the total cost to acquire the required items. The main results include: (1) a problem formulation for the combinatorial reverse auction problem; (2) a solution methodology based on Lagrangian relaxation and (3) analysis of numerical results based on our solution algorithms. By applying Lagrangian relaxation technique, the original optimization can be decomposed into a number of bidders' subproblems which can be solved efficiently. Analysis of the numerical results shows that our algorithm can generate near-optimal solution within acceptable CPU time.

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