# Assessing Patent Value by Gray Relational Analysis and Extension Neural Network

## -A Case Study of Patent Infringement Lawsuits-

Hui-Chung Che[1], Chung-Hsing Peng[2]

[1] *IIUnion IP Consulting Ltd.*
*Rm1701, Tower F, Wanda Square, Shijingshan District, Beijing 100124, P.R. China*
[2] *Gainia Intellectual Asset Services, Inc.*
*Rm219, Bldg.53, No.195, Sec.4, Chunghsing Rd., Chutung, Hsinchu, Taiwan 31040, ROC*
imcharlie@gainia.com
gracepeng@gainia.com

*Abstract*—**The patent became the most important Technology outcome in the era of knowledge economics. This study proposed an intellectual valuation model for monetary patent values on the basis of patent law. The damage award of a patent infringement lawsuit was deemed to be the monetary value of a patent. 65 Effective samples were extracted from 4,289 patent related lawsuits which were retrieved in U.S. district courts of Delaware, California and Texas. 17 patent indicators were defined to describe quantitative features of patents. The Gray Relational analysis was applied to discuss 17 patent indicators' relational grade relating to the damage award. Consequently 10 selected patent indicators with higher Gray Relational coefficients were selected. The Extension Neural Network was then applied to construct the patent valuation model, wherein 10 selected patent indicators were the inputs and the damage award was the output. The proposed patent valuation model was validated to have the predictive power by error analysis. It accommodated to valuate the possible damage award in disputing patent infringement lawsuits. It also contributed to settlement negotiation, patent transaction, patent licensing, etc.**

*Keywords*—**Patent Valuation; Gray Relational analysis; Extension Neural Network; Damage Award; Infringement Lawsuit**

## 1. INTRODUCTION

As technologies develop rapidly and the era of knowledge economics arises, intangible assets show their higher significance than before. The patent stands for a leading role among various species of intangible assets. The patent contributes to enterprises by revenue, stock performance, reputation, research and development, so as to be an important factor for evaluating enterprises and nations in aspects of operation, management and innovation.

However, when considering the patent value, especially the monetary value, it is hard to valuate the patent because the patent is not only a kind of intangible assets, but also a kind of rights. When thinking about the asset, the financial experts usually concentrate their attention on patent's financial contribution. This contribution, such like stock performance or market success, is not directly generated by patents; it is only partly influenced by patents. When thinking about the right, the legal researchers always focus on the scope of patent right and related legal behaviors.

There exists an important phenomenon recently, that is, patent infringement lawsuits grow distinctly around the world. Damage award, licensing fee, and royalty become conspicuous parts of income, and even turn into the majority of revenue in some new start-up companies. No matter in negotiations of patent licensing, patent transaction, hypothecation of intangible assets, or shareholding by patent-based technologies, the monetary value of the patent is always a critical issue. Meanwhile, a reasonable and reliable patent valuation model is always discussed seriously for making patents become monetary assets. The issue of monetary valuation of patents is concerned by people including employees, chiefs, investors, researchers, and professionals among fields of technology management, financial operation, legal strategy, and business administration.

The existing patent valuation models in practice might be briefly summarized to three approaches, such as the cost-based approach, the market-based approach, and the revenue-based approach. These approaches originated basically from the financial methods and then been modified.

The basic idea, on which the cost-based approach is based, is the idea of replacement. This means the value of a patent is identified as the amount that would be necessary to replace the protection right or the related economic benefit potential. The logic behind this approach is that a prospective buyer acting in a logical way would not be willing to pay more for a patent than the amount he would have to pay to obtain an equivalent protection right. The costs compared could be, for example, historical costs, costs of replacement or costs of reproduction, depending on the valuation method used. One advantage of the cost approach is that the evaluator of patents has little influence on the valuation result.

The market-based approach is based on a comparison with a corresponding transaction between independent third parties. That is, the value of a patent is defined through comparison to a similar patent, the market price of which is known through an earlier purchase or sale. In this market, there has to be a sufficient number of comparative transactions in the recent past, for which the obtained retail price is known. If this information exists, the market approach is easy to apply and leads to a valuation result that is acceptable and easy to comprehend. But the prerequisite of an active market is hardly met for patents. Furthermore, the published licensing rates are not sufficient for an adequate comparison.

The basis of the revenue-based approach is the comparison of the future economic benefit of a patent with the future benefit of an alternative investment. So far, the revenue-based approach implements the definition of value most directly. With the application of the revenue-based approach, the sum of advantages, i.e. the additional returns or saved costs less accruing costs, that will arise from the patent will be ascertained. These economic benefits are compared to the best alternative investment, which shows the same future payment flows and the same investment risk. With respect to the valuation, the comparison is made by determining the future economic benefit of the protection right and then discounting it with a risk-adapted interest rate to its actual cash value. To put it another way, the revenue-based approach answers the question: what sum would have to be invested in another way to achieve identical payment flows with the same risk? The valuation results would be somehow risky since the data employed are only prediction-based values and cannot be determined with certainty.

Unfortunately, the aforementioned financial approaches for patent valuation usually disregard the subject matter of the patent and species of enforcement defined and restricted by patent law. These existing patent valuation approaches are more likely named as the “Technology” valuation approaches. It has to be emphasized that a vital difference exists in the scope of right between a technology and a patent. The vital difference will result in different valuation outcome. The right of a technology is knowledge-based power to make, use, or sell; whereas the right of a patent is the power to exclude others from making, using, selling or importing.

Regarding the topics involved the patent law with the patent value, Pakes and Putnam [18] used the cost of patent prosecution as the indicator to evaluate patents. Lanjouw [16] discussed the behaviors in patent prosecution for evaluating patents. Lanjouw and Schankerman [17] discussed the behaviors in patent infringement lawsuits for evaluating patents. Reitzig, Henkel and Heath [20] proposed that the patent infringement lawsuit affected the firm’s strategies.

According to U.S. patent law 35U.S.C.154 ”*Every patent shall contain a short title of the invention and a grant to the patentee, his heirs or assigns, of the right to exclude others from making, using, offering for sale, or selling the invention throughout the United States or importing the invention into the United States, and, if the invention is a process, of the right to exclude others from using, offering for sale or selling throughout the United States, or importing into the United States, products made by that process, referring to the specification for the particulars thereof.*” the right of a patent for the patentee is definitely designated for excluding others from five species of unauthorized behaviors: making, using, offering for sale, selling and importing. Based on the concept of patent law, any patent which can not be enforced the right to exclude others from aforementioned five behaviors would be regarded as legally valueless. The existing patent valuation models in practice usually take this important legal issue aside.

As described above, it's therefore a principal objective of this study to rediscover the patent value in view of patent law by investigating patent infringement lawsuits because the documents of patent infringement lawsuits indicate patents and their momentous, direct and monetary patent values, i.e. damage awards.

It's another objective of this study to construct a monetary valuation model of patents by discussing the mathematical relationship between damage awards and patents.

## 2. REVIEW OF LITERATURES

Regarding the topic of patent valuation and patent indicators, Cockburn and Griliches [3] first discussed the relationship between stock performance and patents. Albert, Avery, Narin and McAllister [1] applied the citation count as the indicator to evaluate patents. Tong and Frame [23] used the patent claim as the indicator to evaluate national technology outcome. Hirschey and Richardson [10] suggested that scientific measures of the quality of inventive output were useful indicators to investors. In this literature, the scientific measures of the quality meant the prior arts of non-patent references of patents. Hereof, Schererc and Vopel [8] suggested that the number of prior arts and citations received were related positively to the patent value; non-patent references were informative about the value of pharmaceutical and chemical patents, but not in other technical fields; patents, which were upheld in opposition and annulment procedures, and patents representing large worldwide patent families were particularly valuable. In this literature, backward citations, forward citations, non-patent references, and worldwide patent families were concluded to be positive to the values of the patent. Hirschey and Richardson [11] found a favorable stock-price influence when both the number of patents, the scientific merit of these patents, and the R&D spending were high, where patent citation information could indeed help investors judge the future profit-earning potential of a firm's scientific discoveries. In this literature, backward citations, forward citation, and non-patent references were concluded to be positive to the stock-price. Reitzig [19] inspected almost all the possible detailed patent indicators with the market value of the patent owner. He concluded that actions of the prosecution were positive to the market value of the patent owner. But legal values of patents in this literature were not considered. Hall, Jaffe and Trajtenberg [7] used the patent citation count as the indicator and discussed its contribution to market value. Von Wartburg, Teichert and Rost [24] proposed a methodological reflection and application of multi-stage patent citation analysis for the measurement of inventive progress. In this literature, backward citations and forward citations were concluded to be positive to R&D activities. Choy, Kim and Park [2] employed patent analysis in cross-impact analysis of syntheses and interactions between various technologies and expected to help practitioners to forecast future trends and to develop better R&D strategies. In this literature, influences of patents were thoroughly analyzed, but legal values of patents were ignored. Hereof and Hoisl [9] described the characteristics of the German Employees' Inventions Act and discussed which incentives it created with a survey of 3,350 German inventors to test the hypotheses regarding this institution; and finally concluded that the law created substantial monetary rewards for productive inventors. In this literature, the patent law was watched and the law-related value of patents was discussed. Silverberg and Verspagenb [21] focused on the analysis of size distributions of innovations by using patent citations as one indicator of innovation significance. In this literature, backward citations, forward citations, and non-patent references were concluded to be positive to innovations but legal values of patents were not discussed. Van Trieste and Vis [24] focused on evaluating a patent on the basis of cost-reducing process improvements from the viewpoint of the patent-holding firm by considering the relevant cash flows that resulted from owning the patent, wherein the patent value was determined by licensing fees, royalty income, and the competitive advantage resulting from the patent and patent maintenance costs. In this literature, the law-related value of patents was first discussed, but no relationship was found between this law-related value and patent indicators.

The aforementioned literatures discussed lots of patent indicators and their contributions such like market success and stock performance. However, such contributions are not directly generated by patents, but they are influenced by patents. Besides, the aforementioned literatures somehow missed an important issue, that is, the patent is a right which given by law. It's more rational to discuss the patent value in view of patent law.

Regarding the topics involved the patent law, Pakes and Putnam [18] used the cost of patent prosecution as the indicator to evaluate patents. Lanjouw [16] discussed the behaviors in patent prosecution for evaluating patents. Lanjouw and Schankerman [17] discussed the behaviors in patent infringement lawsuits for evaluating patents. Reitzig, Henkel and Heath [20] proposed that the patent infringement lawsuit affected the firm's strategies.

Though these literatures discussed patents and indicators in view of patent law, there was no corresponding valuation model built yet.

Referring to the issue of a patent's monetary value, Deng, etc. [6], Thomas [22] tried to use multi-regression to model patent indicators and the stock performance. Unfortunately, the R2 values were too low to explain few. Park and Park [18] proposed a valuation method that generated monetary value, rather than a score or index, based on the structural relationship between technology factors and market factors. This method of generating the patent's monetary value was more useful in practice than the other indicator-based valuation models. Unfortunately, this method was not in view of patent law.

Lai and Che [12, 13, 14], Che, Lai and wang [27] summarized 17 patent indicators for describing quantitative dimensions of patents and applied factor analysis, regressions, neural networks to construct the patent valuation model via patent infringement lawsuits. These literatures advanced the development of patent valuation.

# 3. METHODOLOGY

## 3.1. Population and Sample

This study focuses on the patent infringement lawsuits in U.S. district courts of Delaware, California and Texas. The patent law in U.S. is a federal law, actions for patent infringement filed in federal district courts. Either plaintiff or defendant can appeal to U.S. Court of Appeals for the Federal Circuit (CAFC) if either party does not agree with the judgment determination of district court. By studying a long time in this study, it is found that the lawsuit documents including patent damage award are usually disclosed, discussed, and determined in federal district courts, while only legal topics and questions of law are argued in CAFC. Hence, the patent lawsuits are retrieved through federal district courts in this study. U.S. patent lawsuits are filed in quantity in every federal district. For example, there is up to 2,865 patent related lawsuits from 1944 to 2006 in district court of Delaware, for each lawsuit possesses more than 5,000 documents of miscellaneous issues involved. In order to set up an effective way of modeling, this study only directs to three district courts those are famous in huge quantity and fast judgment of patent infringement lawsuits, i.e. district court of Delaware, district court of California, and district court of Texas. These courts have the accelerated timetable strictly adhered to deadlines, resulting in speedy disposition.

Effective samples for establishing valuation model are reviewed and extracted from all the retrieved patent related lawsuits by meeting all the following criteria:

- Lawsuit terminated to have the final judgment determination in district courts, whereas settled lawsuits excluded.
- Lawsuit disclosed definite patent numbers.
- Lawsuit indicated definite monetary value of damage award.
- The patent involved in a lawsuit possessed all well-defined patent indicators. If a patent infringement lawsuit is too old so that any one of the involved patents does not possess all well-defined patent indicators, such lawsuit should be excluded from effective samples.

Patent infringement lawsuits are retrieved in the period of 1944 to 2006 in both district courts of Delaware and California. But because district court of Texas is famous in showing favor to plaintiffs, lots of lawsuits get settlements, few lawsuits with final judgment of determination are found. Hence, patent infringement lawsuits of district court of Texas are retrieved from 1994 to 2006.

## 3.2. Instrumentation

The database for retrieving patent infringement lawsuits is the LexisNexis. The LexisNexis database originated since 1966 for being developed to the first full text retrieval system of legislation in the world by the American Air Force. It is one of the great law resources in the world, comprising legal documents, industry information, financial documents and public news of all levels of U.S. courts, newspapers, magazines and commercial periodicals.

Samples of lawsuits distribute in different years. The damage award of each lawsuit must be converted to a standard foothold to eliminate the currency revaluation and inflation for consistency of analysis. In this study, the annual interest announced by Federal Reserve System (FED) at the end of each fiscal year is used to convert each damage award to the corresponding value in 2006 by compound interest via engineering economic approach.

By Kolmogorov-Smirnov test, the values of damage awards of all lawsuits are converted by natural logarithm in order to have an approximate normal distribution.

For quantitatively describing properties of the patent, all related previous literatures are thoroughly studied. The quantitative properties of a specific patent, discussed in relevant previous studies were always focused on citations and patent family. Reitzig [19] proposed a brand new indicator: actions of prosecution. Besides, based on the view point of patent law, throughout the opinion of court in patent infringement lawsuits in U.S., a product or a method infringes a patent claim if that product satisfies each of the claim requirements, hence what is claimed is recognized as the invisible boundary of a patent right. Besides, the fewer elements in a claim form a wider boundary, contrarily the more elements in a claim form a narrower boundary. In a patent, an independent claim usually comprises fewer elements, while a dependent claim certainly comprises more elements than the claim being dependent upon. Independent claims are more important than dependent claims, it's therefore not only the number of claims, but also the number of independent claims is considered in this study. International patent Classification (IPC) and U.S. Patent Classification (USPC) are systems for organizing patents. A patent is designated its IPC and USPC by examiners in patent office. The number of IPC and USPC are considered in this study. In addition, according to U.S. patent rule §1.75 (d) (1), the claimed elements and characteristics thereof must be supported by descriptions and drawings, so the number of drawings is also considered. By reviewing previous studies and authors' empirical information and experiences of patent prosecution, patent search, and infringement analysis, there are 17 patent indicators finally summarized.

For normalization of all patent indicators to overcome the high variance, the z-score transformation is applied.

For discussing the relationship between 17 patent indicators and the damage award, both the Multi-Regression analysis and the Gray Relational analysis are applied and compared. Finally, 10 selected patent indicators which having higher Gray Relational coefficients are selected.

The Gray System theory which developed by Deng [4] has been applied in many fields, such as economics, agriculture, geography, weather, earthquakes, etc. The Gray Relational analysis based on the Gray System theory has some advantages: it involves simple calculations and requires a small number of samples; a normal distribution of samples is not needed; the qualified outcomes from the Gray Relational grade do not result in contradictory conclusions from the quantitative analysis; the Gray Relational Grade model is a transfer functional model that is effective in dealing with discrete data [5].

For modeling the nonlinear relationship between 10 selected patent indicators and the damage award, a novel Artificial Neural Networks called Extension Neural Network (ENN) is applied.

Basically, the Artificial Neural Network (ANN) is an information processing paradigm that is inspired by the way biological nervous systems. Learning in biological nervous systems involves adjustments to the synaptic connections that exist between the neurons. The key element of the ANN is the novel structure of the information processing system. It is composed of a large number of highly interconnected processing elements (neurons) working in unison to solve specific problems. The ANN, like human, learns by examples, and is usually conFig.d for some specific applications, such as pattern recognition or data classification. An important issue in ANN design is determining the number of hidden neurons best used in the network. If the hidden number of neurons is increased too much, overtraining will result in the network being unable to "generalize". The training set of data will be memorized, making the network effectively useless on new data sets.

The Extension Neural Network (ENN) was proposed by Wang [26]. This ENN is a combination of the extension theory and the neural network. It uses an extension distance to measure the similarity between data and cluster center. The learning speed of the ENN is faster than the traditional ANN and other fuzzy classification methods. In some applications, the ENN has high accuracy and less memory consumption.

The reason of applying the neural network in this study to modeling the nonlinear relationship between selected factors and the damage award is that, the damage award in any patent infringement lawsuit was first proposed by both parties of plaintiff and defendant, then was discussed, argued, adjusted, and finally determined by the judge or the jury of court. The process for finalizing the damage award is quite humanly and nonlinear, so that the damage award resulted from its corresponding patents is suitably modeled by the neural network.

The input variables for the ENN in this study are 10 selected patent indicators which derived by Gray Relational analysis, while the output variable is the damage award of each lawsuit. For constructing the ENN, basically at least two sets of samples are necessary, i.e. a training set and a testing set, for iteratively tuning the ENN by training and testing. Preferably, for validating the constructed ENN to check its predictive power, another validating set is introduced into the constructed ENN. Various parameters could be tuned in constructing the ENN. In this study, the convergence of Root-Mean-Squared-Error (RMSE) is observed when training, testing and validating the ENN, and therefore regarded as the performance index of the ENN.

### 3.3. Delimitation and Limitation

There are several categories of U.S. patents, such as utility, design, plant, defensive publication, statutory invention registration, and additional improvement, etc. The compositions of all these categories differ from each other. This study discusses the utility patent only because the utility patent plays the major part of all U.S. patent categories. The infringement lawsuits of utility patents are much more than the others.

There is at least one patent included in a patent portfolio which is enforced in a patent infringement lawsuit to win a lump sum of the damage award. Only damage award of the portfolio and portfolio-based patent indicators are analyzed and modeled. This study doesn't probe into the specific value of the specific patent in a portfolio nor discuss the contribution of any specific patent indicator of any specific patent in a portfolio.

Only patent infringement lawsuits with final judgment of determination are analyzed. Actually, settlements exist in lots of patent infringement lawsuits for terminating the suits because either the defendant might want to reduce possible damage award or the plaintiff might want to reduce the litigation cost. In settled lawsuits, neither final judgment determination nor damage awards are found; such settled lawsuits should be excluded in this study.

Only quantitative patent indicators are analyzed. Qualitative patent indicators, such as patent quality and patent enforceability are excluded.

## 4. Analysis and Result

### 4.1. Effective Samples

By using the searching keyword "patent" in the LexisNexis database, 4,286 patent related lawsuits are searched from district courts of Delaware, California and Texas. However, not all of these lawsuits are infringement involved, further searching is needed. Thereby, the searching keyword selected from the group consisting of "damage" and "$" is then applied. The retrieved documents are carefully reviewed and checked by manpower; finally 65 effective samples (lawsuits) including 163 patents are derived, as shown in Table 1.

There are 37 samples including 103 patents are in district court of Delaware; 24 samples including 52 patents are in district court of California; and 4 samples including 8 patents are in district court of Texas. In each of these effective samples, the damage award is clearly indicated, and the patent(s) involved has all 17 patent indicators. Once the infringement lawsuit in which patent involved is too old to be short of some patent indicators, the infringement lawsuit is then discarded.

**TABLE 1**
**SAMPLES RETRIEVED AND EXTRACTED FROM DISTRICT COURTS**

| Lawsuit resource | | D.C. of Delaware | D.C. of California | D.C. of Texas | Sum |
|---|---|---|---|---|---|
| Lawsuits retrieved | | 2,865 | 1,314 | 110 | 4,289 |
| Lawsuits extracted | Lawsuits | 37 | 24 | 4 | 65 |
| | Patents | 103 | 52 | 8 | 163 |

**TABLE 2**
**PATENT INFRINGEMENT LAWSUITS IN EACH YEAR**

| Year | Lawsuits | Year | Lawsuits |
|---|---|---|---|
| 2006 | 12 | 1997 | 2 |
| 2005 | 7 | 1996 | 0 |
| 2004 | 11 | 1995 | 1 |
| 2003 | 5 | 1994 | 0 |
| 2002 | 6 | 1993 | 0 |

| | | | |
|---|---|---|---|
| 2001 | 4 | 1992 | 1 |
| 2000 | 3 | 1991 | 2 |
| 1999 | 3 | 1990 | 0 |
| 1998 | 6 | 1989 | 2 |

In the 65 effective samples, the portfolio size in a lawsuit varies from one patent to 17 patents; the damage award varies from USD 470,084 to USD 2,600,000,000. Table 2 shows the counts of infringement lawsuits from 1989 to 2006. Obviously, infringement lawsuits after 2000 are much more than those before 2000. Since lawsuits with final determinations are only a small part of all infringement lawsuits. The information in Table 2 reveals that patent infringement lawsuit gradually becomes a kind of business in 21 century.

## 4.2. Patent Indicators

Lai and Che [12, 13, 14], Che, Lai and wang [27] proposed 17 patent indicators to describe quantitative features of patents. This study follows. The 17 patent indicators are shown below and summarized in Table 3.

$X_1$ named "Assignees" is the count of assignees of each patent.

$X_2$ named "Inventors" is the count of inventors of each patent.

$X_3$ named "Total claims" is the count of total claims of each patent.

$X_4$ named "Independent claims" is the count of independent claims of each patent. Total claims comprise independent claims and dependent claims. $X_4$ is a part, but the most important part of $X_5$.

$X_5$ named "US patent references" is the count of US patent documents listed in the field of "References Cited", i.e. prior arts recognized by the examiner, of each patent. Some literatures called "US patent references" as the "backward citations".

$X_6$ named "Foreign patent references" is the count of foreign patent documents in the field of "References Cited" of each patent.

$X_7$ named "Non-patent references" is the count of other publications (non-patent literatures, including papers, handbooks and magazines, etc.) in the field of "References Cited" of each patent. Some literatures called "Non-patent references" as the "science linkage".

$X_8$ named "Forward citations" is the count of citations by the other patents in the beginning of lawsuit of each patent.

$X_9$ named "International Patent Classifications (IPC)" is the count of IPCs recognized by the examiner of each patent.

$X_{10}$ named "US Patent Classifications "is the count of USPCs recognized by the examiner of each patent.

$X_{11}$ named "Worldwide patent families" is the count of worldwide related patents those claimed at least one same priority of each patent. This count is investigated based on INPADOC database.

$X_{12}$ named "US patent families" is the count of US related patents those claimed at least one same priority of each patent. This count is investigated based on INPADOC database.

$X_{13}$ named "Office actions" is the count of office opinions by the examiner of USPTO of each patent. The office opinions include the selection by restriction, non-final rejection, final rejection, and notice of allowance, etc.

$X_{14}$ named "Responses" is the count of responses to USPTO by the assignee of each patent. The responses include amendments, response to non-final rejection, response to final rejection, request for continued examination, and appear, etc.

$X_{15}$ named "Examination" is the time span from filing date to issue date of each patent.

$X_{16}$ named "Drawings" is the count of drawings of each patent.

$X_{17}$ named "Life-span" is the time span from filing date to the beginning of lawsuit of each patent.

**TABLE 3**
**PATENT INDICATORS**

| Patent Indicator | |
|---|---|
| $X_1$ | Assignees |
| $X_2$ | Inventors |
| $X_3$ | Total claims |
| $X_4$ | Independent claims |
| $X_5$ | US patent references |
| $X_6$ | Foreign patent references |
| $X_7$ | Non-patent references |
| $X_8$ | Forward citations |
| $X_9$ | International Patent Classifications (IPC) |
| $X_{10}$ | US Patent Classifications |
| $X_{11}$ | Worldwide patent family |
| $X_{12}$ | US patent family |
| $X_{13}$ | Office actions |
| $X_{14}$ | Responses |
| $X_{15}$ | Examination |
| $X_{16}$ | Drawing |
| $X_{17}$ | Life-span |

## 4.3. Descriptive Analysis

For analyzing all patent indicators, the descriptive statistics of the mean, the standard deviation, the maximum and the minimum of each portfolio-based patent indicator is performed as shown in Table 4. Because these variables $X_1$ to $X_{17}$ do not have the same unit for counting, the averages and the standard deviations of all these variables differ critically.

In Table 4, $X_3$ (Total claims), $X_5$ (US patent references), $X_7$ (Non-patent references), $X_8$ (Forward citations), $X_{11}$ (Worldwide patent families), and $X_{12}$ (US patent families) have higher means and standard deviations than the others. The high variances of all these patent indicators $X_1$ to $X_{17}$ will ruin any mathematic model. In order to improve the consistency of analysis, the normalization of all the 17 patent indicators ($X_1$ to $X_{17}$) is necessary. It is therefore to make the 17 patent indicators being transformed to the Z-scores before the following analysis.

**TABLE 4**
**DESCRIPTIVE STATISTICS OF PATENT INDICATORS**

| P.I. | Mean | S.D. | Max. | Min. | S. | Kurt |
|---|---|---|---|---|---|---|
| $X_1$ | 2.4000 | 1.7302 | 9.0 | 1.0 | 1.54 | 2.38 |
| $X_2$ | 5.2615 | 4.6579 | 28.0 | 1.0 | 2.24 | 7.74 |
| $X_3$ | 61.9231 | 66.8104 | 252.0 | 2.0 | 1.54 | 1.49 |
| $X_4$ | 12.4923 | 14.4462 | 72.0 | 1.0 | 2.55 | 7.68 |
| $X_5$ | 51.8769 | 70.0280 | 422.0 | 2.0 | 3.02 | 12.14 |
| $X_6$ | 6.2615 | 9.6926 | 49.0 | 0.0 | 2.31 | 6.19 |
| $X_7$ | 31.5077 | 84.8908 | 568.0 | 0.0 | 4.87 | 27.19 |
| $X_8$ | 41.7385 | 66.6364 | 390.0 | 0.0 | 3.06 | 11.69 |
| $X_9$ | 3.4000 | 2.8218 | 12.0 | 1.0 | 1.61 | 1.87 |
| $X_{10}$ | 9.3538 | 7.9440 | 35.0 | 1.0 | 1.58 | 2.27 |
| $X_{11}$ | 103.2154 | 202.7878 | 1263.0 | 1.0 | 4.04 | 18.85 |
| $X_{12}$ | 36.4154 | 79.6653 | 516.0 | 1.0 | 4.20 | 21.28 |
| $X_{13}$ | 7.5385 | 6.7061 | 28.0 | 1.0 | 1.46 | 1.59 |
| $X_{14}$ | 5.4000 | 5.6397 | 27.0 | 0.0 | 1.68 | 3.07 |
| $X_{15}$ | 5.8531 | 4.5862 | 18.4 | 0.8 | 1.09 | 0.24 |
| $X_{16}$ | 15.9077 | 18.2000 | 79.0 | 0.0 | 1.64 | 2.30 |
| $X_{17}$ | 21.5538 | 20.4122 | 116.0 | 1.0 | 2.08 | 6.36 |

P.I.: Patent Indicator
S.D.: Standard Deviation
S.: Skewness

## 4.4. Regression Analysis

In Regression analysis, if the coefficient of determination $R^2$ value approximates to 1, then the error of the regression model is small and the linear relationship for each indicator corresponding to the damage award is easily explained.

According to the basic idea of Regression analysis, there suggests to have at least 25 samples for each independent variable. For the case of 17 independent variables, 425 samples are required preferably. Since there are only 65 effective samples in this study, the Multi-Regression analysis might fail. It's therefore to have 17 simple linear Regression analyses performed in this study, wherein each normalized patent indicator is the independent variable and the damage award is the dependent variable. Via the tool of SPSS V8.0, the Regression coefficient, $R^2$ and significance for each normalized patent indicator are shown in Table 5.

Though $R^2$ values in these regression analyses are too low to have enough explanatory ability, however, it's still interesting to have some inferences.

**TABLE 5**
**REGRESSION ANALYSIS OF NORMALIZED PATENT INDICATORS**

| N.P.I. | R.C. | $R^2$ | S. |
|---|---|---|---|
| $X_1$ | 0.154 | 0.007 | 0.521 |
| $X_2$ | 0.314 | 0.027 | 0.188 |
| $X_3$ | 0.205 | 0.012 | 0.392 |
| $X_4$ | 0.460 | 0.059 | 0.052 |
| $X_5$ | 0.197 | 0.011 | 0.411 |
| $X_6$ | 0.204 | 0.012 | 0.393 |
| $X_7$ | 0.599 | 0.100 | 0.010* |
| $X_8$ | 0.682 | 0.129 | 0.003** |
| $X_9$ | 0.113 | 0.004 | 0.636 |
| $X_{10}$ | 0.094 | 0.002 | 0.696 |
| $X_{11}$ | -0.094 | 0.002 | 0.696 |
| $X_{12}$ | -0.116 | 0.004 | 0.629 |
| $X_{13}$ | 0.202 | 0.011 | 0.399 |
| $X_{14}$ | 0.230 | 0.015 | 0.336 |
| $X_{15}$ | 0.353 | 0.035 | 0.138 |
| $X_{16}$ | 0.360 | 0.036 | 0.130 |
| $X_{17}$ | 0.342 | 0.033 | 0.150 |

* Significant at 10% level
** Significant at 5% level
N.P.I.: Normalized patent indicator
R.C.: Regression Coefficient
S.: Significance

There are two patent indicators reaching significant level, i.e. $X_7$ (Non-patent references) and $X_8$ (Forward citations), and having higher positive regression coefficients. It means $X_7$ (Non-patent references) and $X_8$ (Forward citations) positively contribute to the patent legal value: the damage ward. Hirschey and Richardson [10], Hereof, etc. [8], Hirschey and Richardson [11], Von Wartburg, etc. [25], and Silverberg and Verspagenb [21] proposed that citations which including backward citations, forward citations, or non-patent references contribute the value of patents. By the Regression analysis, $X_7$ (Non-patent references) and $X_8$ (Forward citations) echo the aforementioned literatures. In addition, $X_5$ (US patent references) which usually called "backward citations" gets positive regression coefficient though it is not significant.

There are two patent indicators with negative regression coefficients, i.e. $X_{11}$ (Worldwide patent families) -0.094 and $X_{12}$ (US patent families) -0.116; the other patent indicators get positive regression coefficients. The negative patent indicators might negatively affect the damage award, whereas the positive ones might contribute the damage award positively.

## 4.5 Gray Relational Analysis

In Table 4, there are 9 patent indicators having Skewness coefficients higher than 2.0 and 5 patent indicators having Skewness coefficients higher than 3.0. Meanwhile, the patent indicators having higher Skewness coefficients also have higher Kurt coefficients. It means that not samples of every patent indicator have a normal distribution. It's therefore to have the Gray Relational analysis here.

For the Gray Relational analysis,

$$\gamma(x_0, x_i)=(\Delta_{min}+\Delta_{max})/(\Delta_{0i}+\Delta_{max}) \quad (1)$$

Where $i=0, 1, 2, \ldots\ldots n;$

$x_0$ is the referenced array;

$x_i$ is the *i-th* specific comparative array;

$x_i(k)$is the *k-th* data in the *i-th* specific comparative array, $k=0, 1, 2,\ldots, m;$

$\Delta_{0i(k)}$ is the absolute value of the difference of $x_0(k)$ and $x_i(k)$;

$\Delta_{min}$ is the minimum value among all $\Delta_{0i(k)}$, for $i=1,\ldots, n, k=1,\ldots, m;$

$\Delta_{max}$ is the maximum value among all $\Delta_{0i(k)}$, for $i=1,\ldots, n, k=1,\ldots, m;$

$\gamma(x_0, x_i)$ is the Gray Relational coefficient between $x_0$ and $x_i$.

In this study, the damage award is set to be the referenced array $x_0$;

17 patent indicators are set to be the comparative array $x_i$, $i=1$ to $17$;

Each comparative array $x_i$ has $m$ data, where $m=65$, because there 65 effective samples.

Via the calculations of Equation (1), each Gray Relational coefficient accompanied with its coefficients rankings bracketed is shown in Table 6. Table 6 also comprises the Result of Regression analysis and $R^2$ ranking bracketed.

The Gray Relational Analysis shows difference result when compared to the Regression analysis. Fig. 1 makes this difference more clear. In Fig. 1, the horizontal axis represents the patent indicator, the left vertical axis represents the Gray Relational coefficient, and the right vertical axis represents the $R^2$. It is easily found only 5 patent indicators having higher Gray Relational coefficients: $X_7$, $X_{16}$, $X_4$, $X_8$ and $X_{17}$, have corresponding higher $R^2$ values. $X_2$, $X_{15}$ and $X_{14}$, have higher $R^2$ values, but their Gray Relational coefficients are low. More particularly, $X_6$ has the highest Gray Relational coefficient, but its $R^2$ value is quite low.

For the result of Gray Relational analysis, $X_7$ (Non-patent references) and $X_8$ (Forward citations) which having higher Gray Relational coefficients, have higher correlation with the damage award. It still echoes previous literatures that citations contribute patent values. ([10], [8], [11], [25], [21])

In previous literature, Hereof, etc. [8] further concluded that worldwide patent families were positive to patent values. In Table 6 and Fig. 1, $X_{11}$ (Worldwide patent families) and $X_{12}$ (US patent families) both get higher Gray Relational coefficients. They also echo Hereof, etc. [8].

Reitzig [19] suggested that applicant's response behavior should be an import indicator for evaluating patents. On the contrary, $X_{13}$ (Office actions), $X_{14}$ (Responses) and $X_{15}$ (Examination) have the lowest three Gray Relational coefficients in Table 6 and Fig. 1. It's somewhat confused, but easily understood. When performing a patent infringement analysis, there is a rule called "File Wrapper Estoppel". Any statement or explanation proposed by the applicant to the patent office during the period of examination shall restrict and even narrow down the claim scope. Sometimes in an infringement lawsuit, accused product will be free of violation in view of the restricted claim of disputing patent by "File Wrapper Estoppel". The longer examination period usually comprises more office actions and more applicants' responses. The more office actions and more applicants' responses, the more evidences for "File Wrapper Estoppel". The more evidences for "File Wrapper Estoppel", the more harmful to win the damage award. That's might be the reason why $X_{13}$ (Office actions), $X_{14}$ (Responses) and $X_{15}$ (Examination) have lower Gray Relational coefficients.

In addition, it is found that the $X_4$ (Independent claims) has higher Gray Relational coefficient. Based on the view point of patent law, a product infringes a patent claim if that product satisfies all of the claim requirements. Therefore, the fewer elements in a claim form a wider boundary. In a patent, an independent claim usually comprises fewer elements, while a dependent claim certainly comprises more elements than the claim being dependent upon. Hence, the independent claim turns into an important indicator of a patent.

**TABLE 6**
**GRAY RATIONAL COEFFICIENTS OF PATENT INDICATORS**

| N.P.I. | G.R.C. | R.C | $R^2$ | S. |
|---|---|---|---|---|
| $X_1$ | 0.916 (10) | 0.154 | 0.007 (13) | 0.521 |
| $X_2$ | 0.912 (12) | 0.314 | 0.027 (7) | 0.188 |
| $X_3$ | 0.912 (13) | 0.205 | 0.012 (9) | 0.392 |
| $X_4$ | 0.917 (7) | 0.460 | 0.059 (3) | 0.052 |
| $X_5$ | 0.919 (4) | 0.197 | 0.011 (11) | 0.411 |
| $X_6$ | 0.925 (1) | 0.204 | 0.012 (10) | 0.393 |
| $X_7$ | 0.921 (2) | 0.599 | 0.100 (2) | 0.010* |
| $X_8$ | 0.917 (8) | 0.682 | 0.129 (1) | 0.003** |
| $X_9$ | 0.913 (11) | 0.113 | 0.004 (14) | 0.636 |
| $X_{10}$ | 0.910 (14) | 0.094 | 0.002 (16) | 0.696 |
| $X_{11}$ | 0.918 (5) | -0.094 | 0.002 (17) | 0.696 |
| $X_{12}$ | 0.919 (3) | -0.116 | 0.004 (15) | 0.629 |
| $X_{13}$ | 0.910 (15) | 0.202 | 0.011 (12) | 0.399 |
| $X_{14}$ | 0.907 (17) | 0.230 | 0.015 (8) | 0.336 |
| $X_{15}$ | 0.910 (16) | 0.353 | 0.035 (5) | 0.138 |
| $X_{16}$ | 0.918 (6) | 0.360 | 0.036 (3) | 0.130 |
| $X_{17}$ | 0.917 (9) | 0.342 | 0.033 (6) | 0.150 |

* Significant at 10% level
** Significant at 5% level
N.P.I.: Normalized patent indicator
G.R.C.: Gray Relational Coefficient
R.C.: Regression Coefficient
S.: Significance

However, it is quite hard to understand why $X_6$ (Foreign patent references) gets the highest Gray Relational coefficient. In U.S., when an applicant files a patent application to patent office, though the applicant has to provide all known prior arts under the responsibility of Information Disclosure Statement, sometimes the applicant won't provide foreign prior arts because the foreign patent database may be not familiar to him. The possible reason might be the responsible patent examiners' contribution. For a patent or patent portfolio to win an infringement lawsuit, it must win the invalidation issue which is always held by the defendant at the same time. When filing an invalidation issue to a disputing patent, the defendant has to provide new prior arts which have not been disclosed in the "References Cites" of said disputing patent. If a disputing patent has lots of prior arts listed in the "References Cites", it makes the defendant very hard to win the invalidation issue. It is therefore that the strong patent having strong patentability gets more prior arts listed in the "References Cites". There is no one but the examiner will try his best to find out all possible prior arts which including US patent references, foreign patent references and non-patent references. In Table 6 and Fig. 1, not only $X_6$ (Foreign patent references), but also $X_5$ (US patent references) and $X_7$ (Non-patent references) have higher Gray Relational coefficients. It somehow supports the aforementioned inference.

In the middle of the broken line of the Gray Relational coefficients in Fig. 1, there exists an obvious drop between $X_1$ and $X_9$. The 7 patent indicators having Gray Relational coefficients lower than $X_1$ are discarded; the other 10 patent indicators are thereby selected, hereinafter called "selected indicators", for building the ENN patent valuation model.

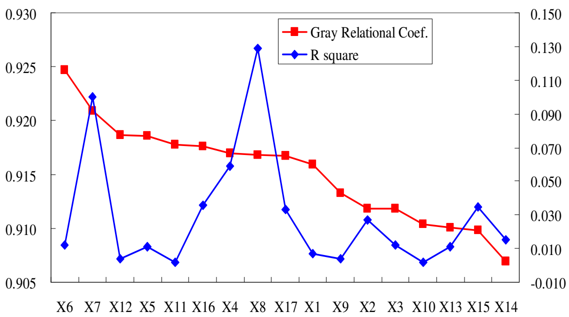


Fig. 1 Screep Plot of Gray Relational Coefficients and R Square

## 4.6 Extionsion Neural Network

The ENN is applied for building the patent valuation. The output variable of the ENN is the damage award. The input variables for the ENN in this study are the 10 selected indicators: $X_1$ (Assignees), $X_4$ (Independent claims), $X_5$ (US patent references), $X_6$ (Foreign patent references), $X_7$ (Non-patent references), $X_8$ (Forward citations), $X_{11}$ (Worldwide patent families), $X_{12}$ (US patent families), $X_{16}$ (Drawings) and $X_{17}$ (Life-span).

Meanwhile, the input variables are normalized to z-scores to be within the interval of 3 times the standard deviation for eliminating the affection of some abnormal values; while the output variable is scaled to 0.2 to 0.8.

Since there are 65 effective samples as shown in Table 1, wherein the 53 samples from 1989 to 2005 are chosen to be the training set and the testing set for constructing the ENN, and the 12 samples in 2006 are chosen to be the validating set for validating the predictive power of the ENN. Moreover, 35 samples are randomly selected from the 53 samples to be the training set and the other 18 samples left are the testing set.

Fig. 2 shows the convergence plot of RMSE versus learning cycle, wherein the vertical axis represents the scaled RMSE, the horizontal axis represents the learning cycle, the upper line represent RMSE of the testing set, and the lower line represents RMSE of the training set. The RMSE value of the training set converges to 0.090 (9.0%) after 2,500 learning cycles and the RMSE value of the testing set converges to 0.118 (11.8%), so the learning process of the ENN is successful. Though these RMSE values are not perfect, they are still acceptable because there are only 53 samples for the learning process. Once the samples increase, the converged RMSE values will decrease.

In the above-constructed ENN, some optimal parameters used are shown below:

- Number of neurons in the first hidden layers: 10
- Number of neurons in the second hidden layers: 1
- Sampling approach for the training set and testing set: random
- Margin for weightings for interconnections: -0.3 to 0.3
- Initial value of the learning speed: 1.0
- Decreasing rate of the learning speed: 0.99
- Initial value of the inertia: 0.5
- Decreasing rate of the inertia: 0.99

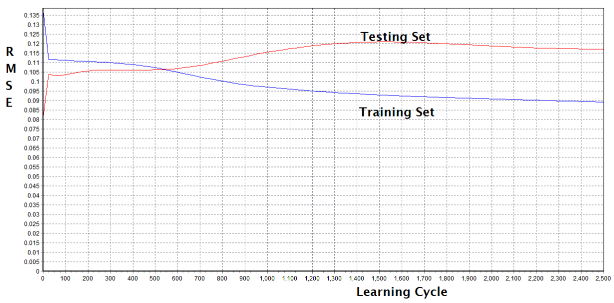


Fig. 2 RMSE Convergence vs. Learning Cycle

For validating the evaluation model, the validating set composed of 12 samples in 2006 is then introduced into the constructed ENN to see its RMSE value and check the predictive power of the constructed ENN. Table 7 shows the comparison of RMSE values of the training set, the testing set and the validating set.

After validation, RMSE 0.096 (9.6%) of the validating set is derived. RMSE 0.096 (9.6%) of the validating set is very close to RMSE 0.090 (9.0%) of the training set and is lower than RMSE 0.118 (11.8%) of the testing set, hence the validation is successful. The successful validation means that the valuation model of the ENN constructed by samples from 1989 to 2005 can predict for samples in 2006. Though the relationship between the damage award and patent indicators is nonlinear, the nonlinear relationship still can be modeled by the ENN with an acceptable error. It proves that the proposed ENN gets the predictive power and the valuation model is feasible. Once a patent or a patent portfolio with its patent indicators is inputted into the ENN valuation model, a possible damage award with an estimated error is outputted.

**TABLE 7**
**RMSE VALUES OF THE TRAINING SET, THE TESTING SET AND THE VALIDATING SET**

| | Number of samples | RMSE |
|---|---|---|
| The training set | 35 | 0.090 (9.0%) |
| The testing set | 18 | 0.118(11.8%) |
| The validating set | 12 | 0.096 (9.6%) |

## 5. DISCUSSION

This study tries to combine the knowledge of patent, finance, computation and management, and to provide a brand new concept for analyzing the patent infringement lawsuits so as to propose a monetary valuation model of patent legal value. This study would like to show that the patent infringement lawsuits are not only good for case study but also good for quantitative analysis.

In this study, 17 patent indicators are proposed for quantitatively describing patents. Z-score transformation is applied to normalize these all patent indicators and to make them be more easily computed. The relationship between the damage award and these proposed 17 patent indicators is first discussed by linear Regression analysis. Only two patent indicators: $X_7$ (Non-patent references) and $X_8$ (Forward citations), are significant and show positive contribution to the damage award. The other patent indicators are not significant. However, via the Regression analysis, it shows that the damage award is not linearly modeled by the patent indicators. The relationship between the damage award and the patent indicators is complicated.

The Gray Relational analysis is applied for resolving the quandary of Regression analysis. $X_7$

(Non-patent references), $X_8$ (Forward citations), $X_{11}$ (Worldwide patent families) and $X_{12}$ (US patent families), which having higher Gray Relational coefficients relating to the damage award, echo lots of previous literatures that citations and patent families contribute patent values. Especially, $X_6$ (Foreign patent references) gets the highest Gray Relational coefficient; whereas $X_{13}$ (Office actions), $X_{14}$ (Responses) and $X_{15}$ (Examination) have low Gray Relational coefficients. These patent indicators need further research.

The Gray Relational analysis provides a new thinking way to help managing and planning the patent strategy.

The nonlinear relationship between the damage award and 10 selected indicators which are selected by the Gray Relational analysis is modeled by the ENN. The valuation model of ENN is constructed via samples from 1989 to 2005 by learning and tuning, and then is validated by samples in 2006. By RMSE analysis between these samples, the proposed patent valuation model shows its predictive power and is proved to be feasible.

It's the first time to successfully discuss the patent legal value in view of patent law, by retrieving samples from patent infringement lawsuits, studying judgments of determination, finding out the patent numbers and damage awards, setting up 17 patent indicators for quantitatively describing patents, discussing the relationship between patent indicators and the damage award, constructing the ENN patent valuation model for modeling patent indicators and the damage award, and finally validating the predictive power of the proposed valuation model.

Fig. 3 shows the architecture of the patent valuation model. For the application in practice, please see the bold lines in Fig. 3, once the 17 patent indicators of a patent or a patent portfolio is described to be inputted in the valuation model as the input variables, consequently the selected indicators are set to be the input variables of the ENN, then an output variable is generated to be the possible value of damage award. Referring to the dotted lines in Fig. 3, the ENN would be certainly improved by feeding more samples of patent infringement lawsuits so as to refine the patent valuation model. These samples could be retrieved from the district courts other than Delaware, California and Texas, More particularly, because the timing issue is already considered in the converted damage awards and the patent indicators: $X_8$ (Forward citations) and $X_{17}$ (Life-span), as years go by, the valuation model learns to adjust itself dynamically. A single patent or a patent portfolio via this model can be valuated to distinct values at different time. It makes sense to make the valuation model to fulfill the reality. It's a live and growing valuation model for providing the monetary legal values of patents.

This proposed valuation model is quite useful in practice and may be widely applied in many fields. For the patent infringement lawsuit, either the plaintiff or the defendant may use the proposed valuation model to estimate the possible damage award earned or lost, so as to configure the optimal lawsuit strategy. For technology management purpose, the R&D intensive company may use the proposed model for valuating the patent assets to distinguish the high value patents from the low value patents. The high value patents could be kept firmly for seeking the chance of lawsuits and strong license. The low value patents might be used for auction, donation or even abandonment. The proposed model also accommodates to applications of patent auction, patent transaction, patent licensing, hypothecation of intangible assets, and shareholding by patent-based technologies, etc.

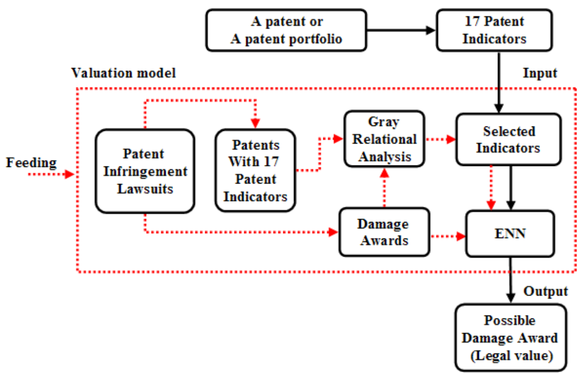


Fig. 3 Architecture of the Proposed Patent Valuation Model

## 6. Recommendation

It is also suggested that some topics might be suitable for further studies:

(1) Variance analysis: It speculates that the proposed evaluation model accommodate to various U.S. district courts, various industries, various technologies, and even various countries. Hence, retrieval of more effective samples and variance analysis are necessary for appropriately adjusting patent indicators and optimal parameter settings of the ENN.

(2) Optimal design for patent compositions for maximizing the damage award: It would be possible by setting the damage award as the object function while all patent indicators as independent variables, so as to get an optimal solution for patent compositions. This would be great helpful to managing R&D outcome, innovation activities, and patent attorneys.

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