A Novel Differentially Coherent PN Code Acquisition Technique for DS-CDMA Mobile Communications

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*Abstract***—Code synchronization is a critical step for fast and reliable code search in directsequence spread-spectrum (DS-SS) system. In this paper, we propose a novel differentially coherent (DC) detection scheme for rapid code acquisition over a fast Rayleigh fading mobile radio channel with frequency offset, and compare the performance with the conventional I-Q noncoherent detection scheme. Numerical results show that the proposed DC scheme outperforms the conventional I-Q noncoherent scheme.**

Keywords— Differentially coherent, PN code Acquisition, synchronization, DS-CDMA, direct sequence spread spectrum (DS-SS)

1. INTRODUCTION

In direct-sequence code-division multipleaccess (DS-CDMA) systems, code synchronization is one of the most important parts, that is usually completed in two steps: acquisition for coarse alignment and tracking for fine alignment [1], the former part is considered in this paper. The objective of code acquisition is to synchronize the received pseudo-noise (PN) code and the local despreading code of receiver within one code chip interval. In the acquisition systems, there are two important goals: to reduce the mean acquisition time and to avoid the probability of false alarm.

Various acquisition schemes have been investigated for rapid acquisition. One way to achieve fast acquisition is to use a double-dwell search scheme. The advantage of double-dwell scheme is the significant reduction of the false alarm, which has two modes of operation: search mode and verification mode. The former is used to make a tentative decision on the received code phase, and the latter is used to verify the decision in the search mode. Therefore, the decision should be effective to avoid false alarm in the verification mode. A simple method to enhance reliability is to increase the number of detections in the verification mode, which is called a majority logic-type decision strategy [2][3].

In order to minimize the code search time, a parallel I-Q noncoherent technique with doubledwell search scheme has been proposed in [4] and [5], where the performance of the parallel and serial I-Q noncoherent techniques have been compared. It has been found that the parallel I-Q noncoherent technique outperforms the serial I-Q noncoherent technique. Nevertheless, it can be seen from [4] and [5] that the performance of the I-Q noncoherent system is very sensitive to the fading rate.

Differential detection has been widely used for the detection of a differential phase-shift keying (DPSK) signal. This technique does not require phase estimation and the phase reference information is obtained from the received signa l preceding the received signal to be detected. Recently, a differentially coherent (DC) PN code acquisition receiver has been proposed in [6] where the down-converted baseband form of the received signal is fed into the PN matched filter (MF) and the following differentially coherent detector. Depending upon the duration of observation, the acquisition detectors can be differentiated according to whether they utilize partial period correlation (PPC) or full period correlation (FPC). With FPC, the correlation is performed over a full code period, whereas the correlation is carried out over a segment of the

Fig. 1 Proposed differentially coherent detector structure.

long PN sequence using PPC. It has been shown that this DC detector is able to reduce the effects of background noise in a static channel without frequency offset. It is notable that the matched filtering is done before differentially detection in this scheme. In fast fading mobile environments where the phase of the received signal fluctuates fast, the performance of this scheme is expected to degrade remarkably. The phase information is lost when the received signals with distinct fluctuating phases sampled at chip rate are summed in the matched filter. The following differential operation at symbol rate could not cancel phase ambiguity since the phase consistency over two consecutive correlation intervals is not guaranteed.

In this paper, we propose a new DC PN code detector where the multiplication of the received signal and the sliding local code is executed first then the result is passed to the differential detector for further processing. The differential detector combined with the integrator is essentially a correlator. This technique performs differential operation at chip rate before matched filtering and thus yields robust acquisition performance in a fast Rayleigh fading channel.

This paper is organized as follows. Section 2 describes the proposed DC detector system model. The performance of the DC detector is analysed in section 3. In section 4, the mean acquisition time in serial and parallel search schemes are calculated using the flow-graph method for DC and I-Q techniques. Numerical results are given in section 5. The section 6 concludes the paper.

2. DC DETECTOR SYSTEM MODEL

The receiver structure for the proposed DC detector system is shown in Fig. 1. It mainly consists of a differential detector, a integrator,

and a logic decision device. This scheme first processes the product of received low-pass filtered signal and local PN code and then performs a complex differential process with one-chip time delay. After integration, if the received and local codes are closely correlated, the threshold η is exceeded and a hit is declared.

The received signal in a flat Rayleigh fading channel can be expressed as

$$
r(t) = \text{Re}\left\{\sqrt{2} \sum_{i=-\infty}^{\infty} \tilde{f}_i c_i p_i (t - \delta T_c) \right\}
$$

$$
\cdot e^{j((\omega_0 + \omega_{of})t + \theta(t))} + n(t) \tag{1}
$$

where T_c is the chip time; ω_0 is the local oscillator radian frequency; ω_{off} is the offset between the received signal and the local radian frequencies; $\theta(t)$ is a slowly-varying carrier phase function which is assumed to be constant over three successive chips; $p_i(t)$ is a unit square pulse from $(i-1)T_c$ to T_c ; and δ is the normalized received code phase offset at $t = 0$; The noise process $n(t)$ is AWGN with one-sided power spectrum density of *N*⁰ W/Hz and is independent of the fading process. The \tilde{f}_i represents the fade statistic for the ith chip which is complex zero mean with correlations $\{\widetilde{f}_i\widetilde{f}_j\}=0$ 2 $\frac{1}{2}E\big\{\widetilde{f}_i\widetilde{f}_j\big\}=0$ and $\frac{1}{2}E\big\{\widetilde{f}_i\widetilde{f}_j\big\}^*\big\}= \rho_{i-j}|\sigma_f|^2$ 2 $\frac{1}{2} E \left\{ \widetilde{f}_i \widetilde{f}_j \right\}^* = \rho |_{i-j} |\sigma_f|^2$, where σ_f^2 is the power of the faded signal and $1 > \rho_1 > \rho_2 > \rho_3 > \dots > 0$ are the correlation coefficients between fading samples. We define the integer *K* as a fading-rate indicator parameter such that $1 >> \rho_{K+1} \simeq 0$.

The integrator output is then sampled every ΔT_c seconds and compared to a threshold η , then decisions are made. Here ΔT_c represents

code search interval with Δ^{-1} a positive integer. Notationally,

$$
Y_k \doteq \frac{1}{N_0 \sqrt{M}} Y(MT_c + k\Delta T_c)
$$

=
$$
\frac{1}{N_0 \sqrt{M}} \int_{k\Delta T_c}^{(M + k\Delta)T_c} \text{Re}\{\tilde{y}(\alpha)\} \cdot c(\alpha - k\Delta T_c - d_1 T_c) d\alpha
$$

$$
Y_k \ge \eta, H_1
$$

$$
< \eta, H_0
$$
 (2)

for $k = 0,1,2,...$, in which the integrator output is normalized by $N_0 \sqrt{M}$. Note that $c(t) = \sum_i c_i p_i(t)$ and $1 < d_1 < L$ is an integer satisfying the shiftand-add property of m-sequences, namely $c_n c_{n-1} = c_{n-d_1}$, $\forall n$. *H*₁ and *H*₀ denote the hypotheses corresponding the in-phase cell and an out-phase cell, respectively.

3 PERFORMANCE ANALYSIS

In the following analysis, we consider a discrete time equivalent of the proposed DC detector. The discrete time system takes samples at the LPF output \tilde{x}_n at a rate of W_{LP} samples/second and all subsequent processing is in the discrete time domain. Since $W_{LP}T_c = N$, there are N samples taken in one chip interval. We let $\delta = n_{\delta} / N$, where the integer n_{δ} is given by $n_{\delta} = Nk_{\delta} + \varepsilon$ with k_{δ} , ε integers and $\varepsilon \in \{0,1,\ldots,N-1\}$. The discrete time integrator output can be expressed as

 ¹ / () 1 0 , * ~ ~ Re *ⁿ ⁿ ^N ⁿ ^N ^k ^d M k N n k N c DT ^k x x c NN M T Y* (3)

where $|x|$ denotes the smallest integer that is greater than x. Let us consider $\Delta = 1$, and define $Y_{DT,k} \doteq Y_{SS,k} + Y_{SN,k} + Y_{NN,k}$, where the signaltimes-signal term *YSS*,*^k* , signal-times-noise term *YSN*,*^k* , and noise-times-noise term *YNN*,*^k* are given by

$$
Y_{SN,k} = \frac{T_c}{NN_0\sqrt{M}} \operatorname{Re} \left\{ \sum_{n=kn}^{(M+k)N-1} \tilde{x}_{S,n} \tilde{x} *_{N,n-N} \cdot c_{\lfloor n/N \rfloor - (k+d_1)} + \tilde{x}_{N,n} \tilde{x} *_{S,n-N} \cdot c_{\lfloor n/N \rfloor - (k+d_1)} \right\}
$$
(4)

$$
Y_{SS,k} = \frac{T_c}{NN_0\sqrt{M}} \text{Re}\left\{\sum_{n=kn}^{(M+k)N-1} \tilde{x}_{S,n}\tilde{x} *_{S,n-N} \cdot c_{\lceil n/N \rceil - (k+d_1)}\right\}
$$

$$
= \frac{T_c}{NN_0\sqrt{M}} \text{Re}\left\{\sum_{n=kn}^{(M+k)N-1} \tilde{f}_{\lceil (n-n_\delta)/N \rceil} \tilde{f} *_{\lceil (n-n_\delta)/N \rceil - 1} \right\}
$$

$$
\cdot e^{j\hat{\omega}_{\text{off}}} c_{\lceil (n-n_\delta)/N \rceil - d_1} c_{\lceil n/N \rceil - (k+d_1)}\right\}
$$
(5)

$$
Y_{NN,k} = \frac{T_c}{NN_0\sqrt{M}} \operatorname{Re} \left\{ \sum_{n=kn}^{(M+k)N-1} \tilde{x}_{N,n} \tilde{x} *_{N,n-N} \right\}
$$

$$
\cdot c_{\lceil n/N \rceil - (k+d_1)} \right\}
$$
(6)

Note that (4) is derived by applying the shift-andadd property and the assumption that $\theta(t)$ is approximately constant over successive chips. Each chip in the sequence $c(t)$ is modeled as an independent random variable with values +1 or - 1 of equal probability. This random modeling of $c(t)$ is appropriate for $M \gg 1$. We assume that the fading process fades much faster than the integrator length MT_c , i.e., $M >> K$. Here we choose $N = W_{LP}T_c = 2$; however, the analysis below also holds for larger values of *N* when the SNR/chip is much smaller than one.

Let $M \gg 1$, then the principle of central-limittheorem can be used to simplify analysis. The *YSS*,*^k* , *YSN*,*^k* and *YNN*,*^k* are treated as approximately Gaussian, which are completely characterized statistically by their first two moments. Since *YSS*,*^k* , *YSN*,*^k* and *YNN*,*^k* are uncorrelated, the mean and variance of $Y_{DT,k}$ can be directly obtained as

$$
E\{Y_{DT,k} | H_i\} = E\{Y_{SS,k} | H_i\}
$$

$$
= \frac{\gamma \rho_1}{N\sqrt{M}} \cos(\hat{\omega}_{off}) \sum_{n=kN}^{(M+k)N-1} 1_{\lceil (n-n_\delta)/N \rceil \lceil n/N \rceil - k}
$$

$$
= \sqrt{M} \gamma \rho_1 \cos(\hat{\omega}_{off}) R_i(p)
$$
 (7)

and

$$
Var{Y_{DT,k} | H_i}
$$

= $Var{Y_{SS,k} | H_i} + Var{Y_{SN,k} | H_i} + Var{Y_{NN,k} | H_i}$
= $\frac{N}{2} + \gamma (1 + \rho_2 \cos(2\hat{\omega}_{off}) R_i(p)) + \frac{1}{2} \gamma^2 G_i(p)$ (8)

Fig. 2 Detection performance comparison of DC detector to I-Q noncoherent detector for varying k; $P_{fa} = 10^{-2}$; M=1024, $\tilde{p} = 0$, $\omega_{eff} = 0$ and 10000 trials for each simulation point.

where $p = k - \delta = (k - k\delta) - \frac{\varepsilon}{N}$ represents the normalized code phase offset between the received and local codes, $\gamma = \frac{f}{N_0}$ $2\sigma_\ell^2$ *N* $\gamma = \frac{2\sigma_f^2 T_c}{N_c}$ the SNR/chip and $\hat{\omega}_{off} = \omega_{off}T_c$. Here we define

$$
R_i(p) = \begin{cases} 1 - |p| & \text{if } |p| < 1 & \text{if } H_1 \\ 0 & \text{otherwise} & \text{if } H_0 \text{ is } H_0 \text{ is } H_1 \text{ is } H_0 \text{ is } H_1 \text{ is } H_0 \text{ is } H_1 \text{ is } H_0 \text{ is } H
$$

and

$$
G_i(p) = \begin{cases} (1-|p|)^2 U, & \text{if } |p| < 1 & (\text{H}_1) \\ (1-|\tilde{p}|) V, & \text{otherwise } (\text{H}_0) \end{cases}
$$
 (10)

with

$$
U = 1 + \rho_1^2 \cos(2\hat{\omega}_{off}) + \frac{2}{M} \sum_{i=1}^{M-1} (M - i)
$$

$$
\cdot \left[\rho_i^2 + \rho_{i+1} \rho_{i-1} \cos(2\hat{\omega}_{off}) \right]
$$
(11)

and

$$
V = 1 + \rho_1^2 (1 + 2\cos(2\hat{\omega}_{\text{off}}))
$$
 (12)

and $\tilde{p} = \varepsilon / N$. The subscript *i* indicates the detector hypothesis: $i=0$ implies the H_0 hypothesis and $i = 1$ implies the H_1 hypothesis. Using the Gaussian assumption for $Y_{DT,k}$, the detection and false alarm probabilities are given by

$$
P_d = \int_{\eta}^{\infty} p_R(x | H_1) dx
$$

=
$$
Q\left(\frac{\eta - E[Y_{DT,k} | H_1]}{\sqrt{\text{var}[Y_{DT,k} | H_1]}}\right)
$$
(13)

and

$$
P_{fa} = \int_{\eta}^{\infty} p_R(y | H_0) dx
$$

=
$$
Q\left(\frac{\eta - E[Y_{DT,k} | H_0]}{\sqrt{\text{var}[Y_{DT,k} | H_0]}}\right)
$$
(14)

where $Q(\cdot)$ denotes the Gaussian Q function. Fig. 2 shows the performance comparison of DC detector to I-Q noncoherent detector. It is shown that the DC detector yields more than 9-10 dB improvement in SNR over the I-Q noncoherent detector.

TABLE1 THE COMPUTATIONAL COMPLEXITY COMPARISON OF TWO DETECTOR SYSTEMS

Detector system	The number of multiplications	The number of additions
Noncoherent I-Q detector	$2(M+1)$	$2(M-1)$
Proposed DC detector	2M	$(M-1)$

(M=integration interval)

Table 1 lists the computational complexity of proposed DC and the I-Q noncoherent detectors. The comparison benchmark is based on the number of multiplication and addition operations needed to complete a code phase search and decision. It is shown that the proposed DC detector needs fewer number of operations than I-Q noncoherent detector, i.e., it has faster execution speed and lower complexity.

4. MEAN ACQUISITION TIME

Let us consider the double-dwell system that has two modes: search mode in first dwell and verification mode in second dwell. If the test samples exceed a threshold in the search mode, then the verification mode will be performed. In the verification mode, the coincidence detection algorithm of [3] is employed (i.e., a majority

logic-type decision strategy) for all systems. The acquisition is declared if B out of A test samples exceed another threshold, or else the system will go back to the search mode until when the acquisition is declared.

The mean acquisition time can be calculated using the flow-graph method in [8] and [9]. Fig. 3 shows the state transition diagram of doubledwell serial search scheme. The state transition diagram of double-dwell parallel search scheme is shown in Fig. 4.

From Fig. 3, it can be shown that

$$
H_D(Z) = P_{d1} Z^{\tau_D} P_{d2} Z^{K\tau_D} \tag{15}
$$

 (15)

$$
H_M(Z) = (1 - P_{d1})Z^{\tau_D} + P_{d1}Z^{\tau_D}(1 - P_{d2})Z^{K\tau_D} \qquad (16)
$$

$$
H_0(Z) = (1 - P_{f1})Z^{\tau_D} + P_{f1}Z^{\tau_D}(1 - P_{f2})Z^{K\tau_D}
$$

$$
+ P_{f1}Z^{\tau_D}P_{f2}Z^{K\tau_D}Z^{P\tau_D}
$$
(17)

In Fig. 4, that parameters are given by

$$
H_1(Z) = P_{d1}Z^T P_{d2}Z^{AT} \tag{18}
$$

$$
H_2(Z) = P_{f1}Z^T P_{f2}Z^{AT} \tag{19}
$$

$$
H_3(Z) = P_{m1}Z^T + P_{d1}Z^T(1 - P_{d2})Z^{AT}
$$

$$
+ P_{f1} Z^T (1 - P_{f2}) Z^{AT} \tag{20}
$$

$$
H_4(Z) = Z^{\mathcal{F}} \tag{21}
$$

where J is the penalty factor. The mean acquisition time for a double-dwell serial search system can be expressed as

$$
E[T_{acq}] = \frac{\partial H(Z)}{\partial Z}\Big|_{Z=1}
$$

=
$$
\frac{\tau_D \Big[1 + KP_{d1} + (q-1)(1 + KP_{f1} + T \times P_F)(1 - \frac{P_D}{2})\Big]}{P_D}
$$
 (22)

where q is the total number of decision samples,

 τ_d is the dwell time, $K = \Delta^{-1}AM$, $T = \Delta^{-1}JM$, $P_D = P_{d1}P_{d2}$ and $P_F = P_{f1}P_{f2}$. Similarly, the mean acquisition time for a double-dwell parallel search system can be expressed as

$$
E[T_{acq}] = \frac{\partial H(Z)}{\partial Z}\Big|_{Z=1}
$$

=
$$
\frac{\left(1 + A(1 - P_{m1}) + JP_F\right)}{NP_D}LT_c
$$
 (23)

Fig. 3 The state transition diagram of doubledwell serial search scheme.

Fig. 4 Simplified state transition diagram of double-dwell parallel search scheme.

5. NUMERICAL RESULTS

In this section, both serial-search and parallelsearch rapid acquisition systems employing the double-dwell DC detection system are compared with I-Q noncoherent detection system based on mean acquisition time performance. We let *N^p* denote the number of parallel branches, each with an integrator of length $M pT_c = \frac{L+1}{Np}T_c$ seconds, in the search detector. We also denote *MT^c* as the integrator length for the serial systems. We use the same assumption for all system schemes, namely, 1) there is only one H_1 cell and $p = 0$, 2) all samples are inde-pendent, 3) $\Delta = 0.5$, 4) $\theta(t) = \theta$, 5) $M_P >> 1$ and $M >> 1$ such that the correlation of the received and local codes yields zero when they are not in-phase, 6) $M_P \gg K$ and $M \gg K$ such that the Markovian nature of the acquisition process is approximately sustained, and 7) the uncertainty region is the full code length *L* . Based on these assumptions, the analytical formulas for evaluating mean acquisition time in (22) and (23) can be directly

Fig. 5 Acquisition performance of serial DC acquisition systems for varying *A* and *B* with $N = 2$, $\Delta = 0.5$, $\tilde{p} = 0$, $\hat{\omega}_{off} = 0$, $J = 10^6 T_c$, and $L = 2^{12} - 1$.

Fig. 6 Acquisition performance comparison of all systems, with $N_p = 8$, $M_p = 512$ for parallel system, $N = 2$, $\rho = 0.8$, $\Delta = 0.5$, $\tilde{p} = 0$, $\hat{\omega}_{off} = 0$, $J = 10^6 T_c$, and $L = 2^{12} - 1$.

applied. The thresholds η_1 and η_2 are selected numerically to minimize the mean acquisition time for each value of SNR/chip.

Fig. 5 illustrates the performance of serial DC system for varying A and B. As the figure indicates, the serial DC system does not benefit when A and B are increased.

Contrary to the DC system, increasing A and B for the parallel I-Q system in the verification mode can obtain better performance.

Fig. 6 illustrates the performance results when the receiver is perfectly aligned with the received signal. It is seen that the proposed DC system has better performance than the I-Q noncoherent system. Fig. 7 illustrates the performance results

Fig. 7 Acquisition performance comparison of all systems, with $N_p = 8$, $M_p = 512$ for parallel system, $N=2$, $\rho=0.8$, $\Delta=0.5$, $\tilde{p}=0$, $\hat{\omega}_{\text{off}} = -\ln \rho$, $J = 10^6 T_c$, and $L = 2^{12} - 1$.

when the receiver is tuned to the transmitted carrier frequency, in this case, the frequency offset is due to Doppler and thus $\left|\hat{\omega}_{off}\right| = -\ln \rho$. In a mobile environment, the MF length *M* can not be increased arbitrarily, therefore, the values of *M* and M_p are assumed to be within the maximum allowed practical limitation.

As both figures indicate, the I-Q system is much more sensitive than the proposed DC system in fading environment. Furthermore, the proposed DC system significantly outperforms the I-Q system when the channel fades fast.

6. CONCLUSIONS

In this paper, a novel DC has been proposed for DS-CDMA code acquisition in a fast Rayleigh fading channel. Numerical results have shown that the DC detector significantly outperforms the conventional I-Q noncoherent detector in the fast varying environments. Specifically, the DC detector yields more than 9- 10 dB improvement in SNR over the I-Q noncoherent detector.

For the computational complexity, the proposed DC detector needs fewer number of multiplication and addition operations than I-Q noncoherent detector, i.e., it has faster execution speed and lower complexity. The ease of hardware implementation is also a remarkable advantage.

REFERENCES

- [1] P.M. Hopkins, "A unified analysis of pseudonoise synchronization by envelope correlation," *IEEE Trans. Commun*., vol. COM-25, pp. 770-778, Aug. 1977.
- [2] A. Polydoros and C.L. Weber, "A Unified Approach to Serial Search Spread-Spectrum Code Acquisition-Part I: General Theory," *IEEE Trans. Commun*., vol. COM-32, pp. 542-549, May. 1984.
- [3] A. Polydoros and C.L. Weber, "A Unified Approach to Serial Search Spread-Spectrum Code Acquisition-Part II: A Matched-Filter Receiver," *IEEE Trans. Commun*., vol. COM-32, pp. 550-560, May. 1984.
- [4] E.A. Sourour and S.C. Gupta, "Direct-Sequence Spread-Spectrum Parallel Acquisition in a Fading Mobile Channel," *IEEE Trans. Commun*., vol. 38, pp. 992-998, July. 1990.
- [5] E.A. Sourour and S.C. Gupta, "Direct-Sequence Spread-Spectrum Parallel Acquisition in Nonselective and Frequency-Selective Rician Fading Channels," *IEEE J. Select. Areas commun*., vol. 10, pp. 535-544, Apr. 1992.
- [6] M.H. Zarrabizadeh and E.S. Sousa, "A Differentially Coherent PN Code Acquisition Receiver for CDMA Systems," *IEEE Trans*. *Commun*., vol. 45, pp. 1456- 1465, Nov. 1997.
- [7] C.D. Chung, "Differentially Coherent Detection Technique for Direct-Sequence Code Acquisition in a Rayleigh Fading Mobile Channel," *IEEE Trans. Commun*., vol. 43, pp. 1116-1126, Feb/Mar/Apr. 1995.
- [8] J.K. Holmes and C.C. Chen, "Acquisition Time Performance of PN Spread-Spectrum Systems," *IEEE Trans. Commun*., vol. COM-25, pp. 778-784, Aug. 1977.
- [9] D.M. Dicarlo and C.L. Weber, "Multiple Dwell Serial Search: Performance and Application to Direct Sequence Code Acquisition," *IEEE Trans. Commun*., vol. COM-31, pp. 650-659, May. 1983.
- [10] J.G. Proakis, *Digital Communications*, 4th ed. New York: Wiley, 2001.