Robust H^∞ Fuzzy Control for Nonlinear Multiple Time-Delay Systems by Dithers:Neural-Network-Based Approach

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***Abstract*—This study presents a robustness design of fuzzy control for nonlinear systems with multiple time delays. First, the neural-network (NN) model is employed to approximate the nonlinear multiple time-delay (NMTD) plant. Then, a linear differential inclusion (LDI) state-space representation is established for the dynamics of the NN model. According to the LDI state-space representation, a robustness design of fuzzy control is proposed to overcome the effect of modeling errors between the NMTD plant and the NN model. Next, in terms of Lyapunov's direct method is proposed to guarantee that the NMTD system under fuzzy control can be stabilized. Subsequently, the stability condition of this criterion is reformulated into a linear matrix inequality (LMI). Based on the LMI, a fuzzy controller is synthesized to stabilize the NMTD system and the H^∞ control performance is achieved at the same time. If the fuzzy controller cannot stabilize the NMTD system, a dither, as an auxiliary of the fuzzy controller, is simultaneously introduced to stabilize the NMTD system. If the frequency of dither is high enough, the trajectory of the dithered system and that of its corresponding mathematical model–the relaxed system can be made as close as desired. This fact enables us to get a rigorous prediction of stability of the dithered system by establishing the stability of the relaxed system.**

Keywords —Neural network (NN), modeling error, H^∞ fuzzy control, dither, time delay.

1. INTRODUCTION

In practice, due to the information transmission, time delays naturally exist in many systems. The existence of time delays is frequently a source of instability and encountered in various engineering systems [1], [2]. Therefore, the problem of stability analysis of time-delay systems has been one of the main concerns of researchers wishing to inspect the properties of such systems. Stability criteria of time-delay systems so far have been approached in two main ways according to the dependence upon the size of delay. One direction is to contrive stability conditions that do not include information on the delay, while the other direction includes methods which take time delay into account. The former case is often referred to as delay-independent criteria and generally gives good algebraic conditions. In particular, some delay-independent stability conditions and stabilization approaches have been proposed for nonlinear time-delay systems. Results are readily available in the literature (e.g. [2]-[8] and the references therein). However, abandonment of information on the size of time delay necessarily causes conservativeness of the criteria, especially when the delay is comparatively small [9].

In the past few years, neural-network (NN)-based modeling has become an active research field because of its unique merits in solving complex nonlinear system identification and control problems (e.g. [10]-[19] and the references therein). Neural networks are composed of simple elements operating in parallel. These elements are inspired by biological nervous systems. As a result, we can train an NN to represent a particular function by adjusting the weights between elements.

Additionally, fuzzy control has attracted a great deal of attention from both the academic and industrial communities during the last decade, and there have been many successful applications [20]-[29]. Despite the success, it has become evident that many basic issues remain to be further addressed. Stability analysis and systematic design are certainly among the most important issues for fuzzy control systems. Lately, there have been significant research efforts devoted to these issues (e.g. [30]-[38]). All of them, however, neglect the modeling error between nonlinear system and fuzzy model. In fact, existence of modeling error may be a potential source of instability for control designs that have been based on the assumption that the fuzzy model exactly matches the nonlinear plant [39]. Recently, Kiriakidis [39], Chen et al. [40]-[41] and Cao et al. [42]-[44] have proposed novel approaches to overcome the influence of modeling error in the field of model-based fuzzy control for nonlinear systems.

Not only the stability but also the control performance of nonlinear systems is important issue for control design. The H^{∞} control problem for nonlinear systems has received considerable attention over the last few decades [40]-[42], [45]-[51]. Hence, a fuzzy control design with guaranteed control performance has been introduced for nonlinear systems in this study. However, to the best of our knowledge, the stabilization problem of robust H^{∞} fuzzy control for nonlinear multiple time-delay systems remains an open area.

Furthermore, it has been long known that the injection of a high frequency signal, known as a dither, into a nonlinear system may improve its performance (e.g. [52]-[63] and the references therein). Better performance is viewed as less distortion in the system output, augmented stability, and quenching of limit cycles as well as jump phenomena [56]. A rigorous analysis of stability in a general nonlinear system with a dither control was given in Steinberg and Kadushin [52]. On the basis of the relaxed method, the relaxed system may be stabilized by regulating appropriately the parameters of dither. Mossaheb [55] pointed out that the dither of sufficiently high frequency may result in output of the relaxed system and that of the dithered system as close as desired. This phenomenon allows us for a rigorous prediction of the stability of the dithered system by establishing the stability of the relaxed system, provided the dither has a high enough frequency. In recent years, there are also some successful applications of dithers, Feeny and Moon [59] applied dither to quench chaos inherent to a stick-slip oscillator and showed that the discontinuity for the low-frequency behavior could be effectively removed. Moreover, Iannelli et al. [61]-[62] indicated that discontinuous nonlinearities of feedback systems could be narrowed using dithers.

However, to our knowledge, making use of dither to overcome the influence of modeling error via neural-network (NN)-based approach has not been discussed yet. A robustness design of H^{∞} fuzzy control for NMTD systems by dithers is hence proposed in this study to improve systems' performance. An NN model is first employed to approximate the NMTD plant. Then, a linear differential inclusion (LDI) state-space representation is established for the dynamics of the NN model. According to the LDI state-space representation, a robustness design of fuzzy control is proposed to overcome the effect of modeling errors between the NMTD plant and the NN model. Next, in terms of Lyapunov's direct method is proposed to ensure that the NMTD system under fuzzy control can be stabilized. Subsequently, the stability condition of this criterion is reformulated into a linear matrix inequality (LMI). Based on the LMI, a fuzzy controller is synthesized to stabilize the NMTD system and the H^{∞} control performance is achieved at the same time. If the fuzzy controller cannot stabilize the NMTD system, the fuzzy controller and the dither (as an auxiliary of the fuzzy controller) are simultaneously introduced to stabilize the NMTD system by regulating the dither's parameters.

2. PRELIMINARY NOTATIONS

The following notations will be used throughout this paper.

N : nonlinear multiple time-delay (NMTD) plant (see (3.1))

$\bar{N}$: closed-loop NMTD system (see Fig. 1a.)

N_d: dithered plant (see (5.1a))

$\bar{N}_d$:closed-loop dithered system (see Fig. 1b.)

N_r: relaxed model of N_d (see (5.1b))

$\bar{N}_r$:closed-loop relaxed system (see Fig. 1c.)

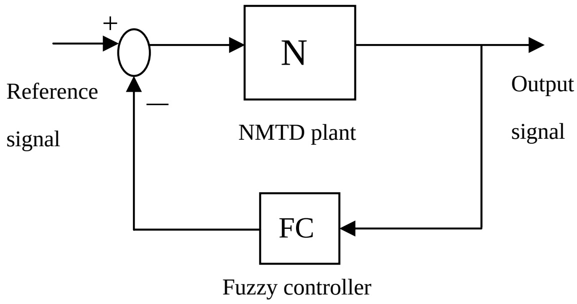


Fig. 1a. Closed-loop NMTD system $\bar{N}$.

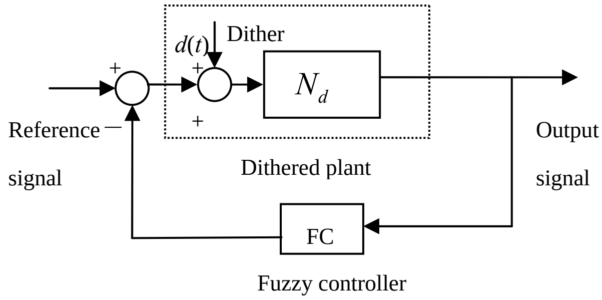


Fig. 1b. Closed-loop dithered system $\bar{N}_d$.

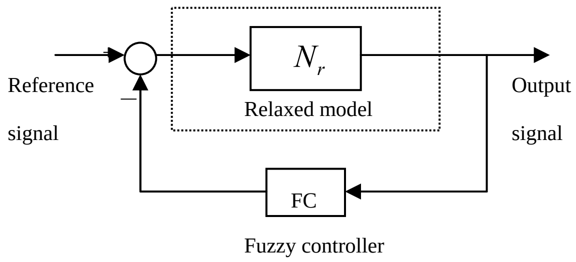


Fig. 1c. Closed-loop relaxed system $\bar{N}_r$.

3. SYSTEM DESCRIPTION

Consider a nonlinear multiple time-delay (NMTD) plant N described by the following equation:

$$N\ :\quad \dot{X}(t)=f(X(t),U(t))+\sum_{k=1}^{L}\rho_k(X(t-\tau_k))+\varpi(t) \qquad (3.1)$$

where $f(\cdot)$ and $\rho_k(\cdot)$ are the nonlinear vector-valued functions which satisfy the assumptions of continuity and boundedness given in [52], $X(t)$ denotes the state vector τ_k, $(k=1,\ 2,\ \cdots,\ L)$ are the time delays and $U(t)$ is a control input vector and $\varpi(t)$ denotes the external disturbance with a known upper bound $\varpi_{ub}(t)\geq\|\varpi(t)\|$.

Definition 3.1 [40]: The solutions of a dynamic system are said to be uniformly ultimately bounded (UUB) if there exist positive constants $\tilde{\varepsilon}$ and $\tilde{\kappa}$, and for every $\hbar \in (0, \tilde{\kappa})$ there is a positive constant $t_1 = t_1(\hbar)$, such that

$$\|x(0)\| < \hbar \ \Rightarrow \ \|x(t)\| \le \tilde{\varepsilon} \quad \forall t \ge t_1. \tag{3.2}$$

In this section, an NN model is first established to approximate the NMTD plant *N*. Then, the dynamics of the NN model is converted into an LDI state-space representation. Finally, a fuzzy controller is synthesized to stabilize the NMTD system.

3.1 Neural-Network (NN) Model

The NMTD plant N is approximated by an NN model, has S layers with $R^{\sigma}\ (\sigma = 1, 2, \cdots, S)$ neurons for each layer, in which $x_1(t) \sim x_\delta(t)$ are the state variables, $x_1(t-\tau_1) \sim x_1(t-\tau_k),\ x_2(t-\tau_1) \sim x_\delta(t-\tau_k)$ are the state variables with delays and $u_1(t) \sim u_z(t)$ are the input variables.

In order to distinguish among these layers, the superscripts are used for identifying the layers. Specifically, we append the number of the layer as a superscript to the names for each of these variables. Thus, the weight matrix for the σ th layer is written as W^{σ}. Moreover, it is assumed that $v_{\varsigma}^{\sigma}(t)(\varsigma = 1, 2, \cdots, R^{\sigma}; \sigma = 1, 2, \cdots, S)$ is the net input and $T(v_{\varsigma}^{\sigma}(t))$ is the transfer function of the neuron. Subsequently, the transfer function vector of the σ th layer can be defined as:

$$\Psi^{\sigma}(v_{\varsigma}^{\sigma}(t)) \equiv [T(v_1^{\sigma}(t))\ T(v_2^{\sigma}(t))\ \cdots\ T(v_{R^{\sigma}}^{\sigma}(t))]^T, \sigma = 1, 2, \cdots, S \tag{3.3}$$

where $T(v_{\varsigma}^{\sigma}(t))(\varsigma = 1, 2, \cdots, R^{\sigma})$ is the transfer function of the ς th neuron. Then the final output of NN model can be inferred as follows:

$$\dot{X}(t) = \Psi^{S}(W^{S}\Psi^{S-1}(W^{S-1}\Psi^{S-2}(\cdots\Psi^{2}(W^{2}\Psi^{1}(W^{1}\Lambda(t)))\cdots))) \tag{3.4}$$

where $\Lambda^T(t) = [X^T(t)\ \ X^T(\mathrm{t}-\tau_{\mathrm{k}})\ \ U^T(\mathrm{t})]$

with $X(t) = [x_1(t)\ x_2(t) \cdots x_\delta(t)]^T$,

$$X(t-\tau_k) = [x_1(t-\tau_1)\cdots x_1(t-\tau_k)\ \ x_2(t-\tau_1)\cdots x_\delta(t-\tau_k)]^T$$

for $k = 1, 2\cdots, L$,

$$U(t) = [u_1(t)\, u_2(t) \cdots u_z(t)]^T.$$

3.2 Linear Differential Inclusion (LDI)

In order to deal with the stability problem of the NMTD system, an LDI state-space representation is established for the dynamics of the NN model and it can be described as [11, 64]:

$$\dot{O}(t) = A(a(t))O(t), A(a(t)) = \sum_{i=1}^{\phi} h_i(a(t))\overline{A}_i \tag{3.5}$$

where ϕ is a positive integer, $a(t)$ is a vector signifying the dependence of $h_i(\cdot)$ on its elements, $\overline{A}_i (i = 1, 2, \cdots, \phi)$ are constant matrices and $O(t) = [o_1(t)\ o_2(t)\ \cdots\ o_{\Xi}(t)]^T$. Furthermore, it is assumed that $h_i(a(t)) \ge 0, \sum_{i=1}^{\phi} h_i(a(t)) = 1$. From the properties of LDI, without loss of generality, we can use $h_i(t)$ instead of $h_i(a(t))$. In the following, a procedure is taken to represent the dynamics of the NN model (3.4) by LDI state-space representation [11].

To begin with, notice that the output $T(v_{\varsigma}^{\sigma}(t))$ satisfies

$$g_{\varsigma 1}^{\sigma} v_{\varsigma}^{\sigma}(t) \le T(v_{\varsigma}^{\sigma}(t)) \le g_{\varsigma 2}^{\sigma} v_{\varsigma}^{\sigma}(t), \qquad v_{\varsigma}^{\sigma}(t) \ge 0$$

$$g_{\varsigma 2}^{\sigma} v_{\varsigma}^{\sigma}(t) \le T(v_{\varsigma}^{\sigma}(t)) \le g_{\varsigma 1}^{\sigma} v_{\varsigma}^{\sigma}(t), \qquad v_{\varsigma}^{\sigma}(t) < 0$$

where $g_{\varsigma 1}^{\sigma}$ and $g_{\varsigma 2}^{\sigma}$ denote the minimum and the maximum of the derivative of $T(v_{\varsigma}^{\sigma}(t))$, respectively, and they are given in the following:

$$g_{\varsigma\phi}^{\sigma} = \begin{cases} \min\limits_{v} \dfrac{d\ T(v_{\varsigma}^{\sigma}(t))}{dv_{\varsigma}^{\sigma}(t)} & \text{when} \quad \phi = 1 \\ \max\limits_{v} \dfrac{d\ T(v_{\varsigma}^{\sigma}(t))}{dv_{\varsigma}^{\sigma}(t)} & \text{when} \quad \phi = 2. \end{cases} \tag{3.6}$$

Subsequently, the min-max matrix G^{σ} of the σ th layer is defined as follows:

$$G^{\sigma} \equiv diag[g^{\sigma}_{\varsigma\ \phi_{\varsigma}}] = \begin{bmatrix} g^{\sigma}_{1\ \phi_1} & 0 & 0 & \cdots & 0 \\ 0 & g^{\sigma}_{2\ \phi_2} & 0 & \ddots & 0 \\ 0 & 0 & g^{\sigma}_{3\ \phi_3} & 0 & \vdots \\ \vdots & \ddots & 0 & \ddots & 0 \\ 0 & 0 & \cdots & 0 & g^{\sigma}_{R^{\sigma}\ \phi_R} \end{bmatrix} \quad (3.7)$$

Moreover, based on the interpolation method, the transfer function $T(v^{\sigma}_{\varsigma}(t))$ can be represented as follows [11]:

$$T(v^{\sigma}_{\varsigma}(t)) = (h^{\sigma}_{\varsigma\ 1}(t)g^{\sigma}_{\varsigma\ 1} + h^{\sigma}_{\varsigma\ 2}(t)g^{\sigma}_{\varsigma\ 2})v^{\sigma}_{\varsigma}(t) = (\sum_{\phi=1}^{2} h^{\sigma}_{\varsigma\ \phi}(t)g^{\sigma}_{\varsigma\ \phi})v^{\sigma}_{\varsigma}(t) \quad (3.8)$$

where the interpolation coefficients $h^{\sigma}_{\varsigma\phi}(t) \in [0,1]$ and $\sum_{\phi=1}^{2} h^{\sigma}_{\varsigma\phi}(t) = 1$. From (3.3) and (3.8), we have

$$\Psi^{\sigma}(v_{\varsigma}(t)) \equiv [T(v^{\ \sigma}_{1}(t)) \quad T(v^{\ \sigma}_{2}(t)) \quad \cdots \quad T(v^{\ \sigma}_{R^{\sigma}}(t))]^{T}$$

$$= [(\sum_{\varphi_1=1}^{2} h^{\sigma}_{1\varphi_1}(t)g^{\sigma}_{1\varphi_1})v^{\sigma}_{1}(t) \ (\sum_{\varphi_2=1}^{2} h^{\sigma}_{2\varphi_2}(t)g^{\sigma}_{2\varphi_2})v^{\sigma}_{2}(t) \ \cdots \ (\sum_{\varphi_R=1}^{2} h^{\sigma}_{R^{\sigma}\ \varphi_R}(t)g^{\sigma}_{R^{\sigma}\ \varphi_R})v^{\sigma}_{R^{\sigma}}(t)]^{T} \quad (3.9)$$

Therefore, the final output of the NN model (3.4) can be reformulated as follows:

$$\dot{X}(t) = \sum_{p=1}^{2} h^{S}_{\varsigma\ p}(t)G^{S}(W^{S}[\cdots[\sum_{n=1}^{2} h^{2}_{\varsigma\ n}(t)G^{2}(W^{2}[\sum_{b=1}^{2} h^{1}_{\varsigma\ b}(t)G^{1}(W^{1}\Lambda(t))])]\cdots])$$
$$= \sum_{p=1}^{2}\cdots\sum_{n=1}^{2}\sum_{b=1}^{2} h^{S}_{\varsigma\ p}(t)\cdots h^{2}_{\varsigma\ n}(t)h^{1}_{\varsigma\ b}(t)G^{S}W^{S}\cdots G^{2}W^{2}G^{1}W^{1}\Lambda(t)$$
$$= \sum_{\Omega} h^{\sigma}_{\varsigma\ \Omega}(t)E^{\sigma}_{\Omega}\ \Lambda(t) \quad (3.10)$$

where

$$\sum_{b=1}^{2} h^{1}_{\varsigma\ b}(t) \equiv \sum_{b_1=1}^{2} h^{1}_{1\ b_1}(t)\sum_{b_2=1}^{2} h^{1}_{2\ b_2}(t)\cdots\sum_{b_R=1}^{2} h^{1}_{R^1\ b_R}(t)$$

$$\sum_{n=1}^{2} h^{2}_{\varsigma\ n}(t) \equiv \sum_{n_1=1}^{2} h^{2}_{1\ n_1}(t)\sum_{n_2=1}^{2} h^{2}_{2\ n_2}(t)\cdots\sum_{n_R=1}^{2} h^{2}_{R^2\ n_R}(t)$$

$$\vdots$$

$$\sum_{p=1}^{2} h^{S}_{\varsigma\ p}(t) \equiv \sum_{p_1=1}^{2} h^{S}_{1\ p_1}(t)\sum_{p_2=1}^{2} h^{S}_{2\ p_2}(t)\cdots\sum_{p_R=1}^{2} h^{S}_{R^S\ p_R}(t)$$

$$\sum_{\Omega} h^{\sigma}_{\varsigma\ \Omega}(t) \equiv \sum_{p=1}^{2}\cdots\sum_{n=1}^{2}\sum_{b=1}^{2} h^{S}_{\varsigma\ p}(t)\cdots h^{2}_{\varsigma\ n}(t)h^{1}_{\varsigma\ b}(t),$$

$\varsigma = 1,\ 2,\ \cdots,\ R^{\sigma}$; $E^{\sigma}_{\Omega} \equiv G^{S}W^{S}\cdots G^{2}W^{2}G^{1}W^{1}$ and b_{ς}, n_{ς}, p_{ς} ($\varsigma = 1,\ 2,\ \cdots,\ R$) represent the variables ϕ of the ςth neuron of the first, second, and the Sth layer, respectively. Finally, according to (3.5), the dynamics of the NN model (3.10) can be rewritten as the following LDI state-space representation:

$$\dot{X}(t) = \sum_{i=1}^{\varphi} h_i(t)E_i\Lambda(t) \quad (3.11)$$

where $h_i(t) \geq 0, \sum_{i=1}^{\phi} h_i(t) = 1$, ϕ is a positive integer and E_i is a constant matrix with appropriate dimension associated with E^{σ}_{Ω}. The LDI state-space representation (3.11) can be further rearranged as follows:

$$\dot{X}(t) = \sum_{i=1}^{\varphi} h_i(t)\{A_iX(t) + B_iU(t) + \sum_{k=1}^{L}\overline{A}_{i\,k}X(t-\tau_k)\} \quad (3.12)$$

where A_i, B_i, and $\overline{A_{ik}}$ are the partitions of E_i corresponding to the partitions of $\Lambda^{T}(t)$.

3.3 Fuzzy Controller

According to the state-feedback control scheme, a fuzzy controller is utilized to stabilize the nonlinear multiple time-delay (NMTD) system. The fuzzy controller takes the following form:

Control Rule j: IF $x_1(t)$ is M_{j1} and $\cdots$ and $x_{\delta}(t)$ is $M_{j\delta}$

THEN $U(t) = -F_jX(t)$

where $j = 1, 2, \cdots, u$, and u is the number of IF-THEN rules of the fuzzy controller and $M_{j\theta}(\theta = 1, 2, \cdots, \delta)$ are the fuzzy sets. Hence, the final output of this fuzzy controller is inferred as follows:

$$U(t) = -\frac{\sum_{j=1}^{\mu} w_j(t)F_jX(t)}{\sum_{j=1}^{\mu} w_j(t)} = -\sum_{j=1}^{\mu} h_j(t)F_jX(t) \quad (3.13)$$

with $w_j(t) \equiv \prod_{\theta=1}^{\delta} M_{j\theta}(x_{\theta}(t))$, $h_j(t) \equiv \frac{w_j(t)}{\sum_{j=1}^{u} w_j(t)}$, in which $M_{j\theta}(x_{\theta}(t))$ is the grade of membership of $x_{\theta}(t)$ in $M_{j\theta}$. In this study, it is also assumed that $w_j(t) \geq 0 (j = 1, 2, \cdots, u)$ and $\sum_{j=1}^{u} w_j(t) > 0$ for all t.

Therefore, $h_j(t) \geq 0$ and $\sum_{i=1}^{\phi} h_i(t) = 1$ for all t.

3.4 H^{∞} Control Design via Fuzzy Control

Stabilizing the closed-loop nonlinear systems and attenuating the influence of the external disturbance $\varpi(t)$ on the state variable $X(t)$ are the objectives of this article. The influence of $\varpi(t)$ will worsen the performance of fuzzy control system. In order to guarantee the control performance by eliminating the influence of $\varpi(t)$ is a significant problem in the control system. Hence, in this work, not only is the stability of fuzzy control system achieved but also the H^{∞} control performance is satisfied as follows:

$$\int_0^{t_f} X^T(t)ZX(t)dt \le X^T(0)PX(0) + \ni^2 \int_0^{t_f} \varpi^T(t)\varpi(t)dt \qquad (3.14)$$

where t_f denotes the terminal time of the control, P is a symmetric positive definite matrix, $\ni$ is a prescribed value which denotes the effect of $\varpi(t)$ on $X(t)$, and Z is a positive definite weighting matrix. The physical meaning of (3.14) is that the effect of $\varpi(t)$ on $X(t)$ must be attenuated below a desired level $\ni$ from the viewpoint of energy [40].

4. ROBUSTNESS DESIGN OF FUZZY CONTROL AND STABILITY ANALYSIS

In this section, the stability of the nonlinear multiple time-delay (NMTD) system is examined under the influence of modeling error.

4.1 Modeling Error

Substituting (3.13) into (3.1) and (3.12) yields the closed-loop NMTD system $\overline{N}$ as follows:

$$\dot{X}(t) = \bar{f}(X(t)) + \sum_{k=1}^{L} \rho_k(X(t-\tau_k)) + \varpi(t)$$

$$= \sum_{i=1}^{\varphi}\sum_{j=1}^{\mu} h_i(t)h_j(t)\{(A_i - B_iF_j)X(t)$$

$$+\sum_{k=1}^{L} \bar{A}_{ik} X(t-\tau_k)\} + \varpi(t) + \Delta\Phi(t) \qquad (4.1)$$

where $\bar{f}(X(t)) \equiv f(X(t),\ U(t))$

with $U(t) = -\sum_{j=1}^{\mu} h_j(t)F_jX(t)$, $\Delta\Phi(t) \equiv e(t) + \sum_{k=1}^{L} e_k(t-\tau_k)$,

in which $e(t) \equiv \bar{f}(X(t)) - \sum_{i=1}^{\varphi}\sum_{j=1}^{\mu} h_i(t)h_j(t)\{(A_i - B_iF_j)X(t)\}$,

$\sum_{k=1}^{L} e_k(t-\tau_k) \equiv \sum_{k=1}^{L} \rho_k(X(t-\tau_k)) - \sum_{i=1}^{\varphi}\sum_{j=1}^{\mu} h_i(t)h_j(t)\{\sum_{k=1}^{L} \bar{A}_{ik} X(t-\tau_k)\}$

,and $\Delta\Phi(t)$ denotes the modeling error between the closed-loop NMTD system (4.1) and the closed-loop NN model [(3.12) and (3.13)].

Suppose that there exists the bounding matrix ΔY_{ij} such that

$$\|\Delta\Phi(t)\| \le \left\| \sum_{i=1}^{\varphi}\sum_{j=1}^{\mu} h_i(t)h_j(t)\Delta Y_{ij} X(t) \right\| \qquad (4.2)$$

for the trajectory $X(t)$, and the bounding matrix ΔY_{ij} can be described as follows:

$$\Delta Y_{ij} = \kappa_{ij} Y \qquad (4.3)$$

where Y is the specified structured bounding matrix and $\|\kappa_{ij}\| \le 1$ for $i = 1, 2, \cdots, \phi$; $j = 1, 2, \cdots, u$.

From (4.2) and (4.3), we have

$$\Delta\Phi^T(t)\Delta\Phi(t) \le [\sum_{i=1}^{\varphi}\sum_{j=1}^{\mu} h_i(t)h_j(t)\Delta Y_{ij}X(t)]^T[\sum_{i=1}^{\varphi}\sum_{j=1}^{\mu} h_i(t)h_j(t)\Delta Y_{ij}X(t)]$$

$$\le \sum_{i=1}^{\varphi}\sum_{j=1}^{\mu} h_i(t)h_j(t)\|YX(t)\|\|\kappa_{ij}\| \sum_{i=1}^{\varphi}\sum_{j=1}^{\mu} h_i(t)h_j(t)\|\kappa_{ij}\|\|YX(t)\|$$

$$\le [YX(t)]^T[YX(t)] \qquad (4.4)$$

Namely, the modeling error $\Delta\Phi(t)$ is bounded by the specified structured bounding matrix Y.

Remark 4.1 [40]: The procedures for determining κ_{ij} and Y are described by the following simple example. Assuming that the possible bounds for all elements in ΔY_{ij} are

$$\Delta Y_{ij} = \begin{bmatrix} \Delta y_{ij}^{11} & \Delta y_{ij}^{12} \\ \Delta y_{ij}^{21} & \Delta y_{ij}^{22} \end{bmatrix} \qquad (4.5)$$

where $-\gamma^{qs} \le \Delta y_{ij}^{qs} \le \gamma^{qs}$ for some γ_{ij}^{qs} with q , s = 1, 2; i =1, 2,…, ϕ; and j =1, 2,…, u.

One possible description for the bounding matrix ΔY_{ij} is

$$\Delta Y_{ij} = \begin{bmatrix} \kappa_{ij}^{11} & 0 \\ 0 & \kappa_{ij}^{22} \end{bmatrix} \begin{bmatrix} \gamma^{11} & \gamma^{12} \\ \gamma^{21} & \gamma^{22} \end{bmatrix} = \kappa_{ij} Y \tag{4.6}$$

where $-1 \le \kappa_{ij}^{qq} \le 1$ for $q = 1, 2$. It is noticed that κ_{ij} can be chosen by other forms as long as $\left\| \kappa_{ij} \right\| \le 1$.

Then, we check the validity of (4.2) in the simulation. If it is not satisfied, we can expand the bounds for all elements in ΔY_{ij} and repeat the design procedures until (4.2) holds.

4.2 Stability in the Presence of Modeling Error

In the following, a stability criterion is proposed to guarantee the stability of the NMTD system $\overline{N}$ described in (4.1). Prior to examination of stability of $\overline{N}$, a useful concept is given below.

Lemma 1 [65, 66]: For any A, B $\in R^n$ and for any symmetric positive definite matrix $G \in R^{n \times n}$ or R, we have

$$-2A^T B \le A^T G A + B^T G^{-1} B.$$

Theorem 1: The NMTD system $\overline{N}$ (4.1) is uniformly ultimately bounded (UUB) and the H^{∞} control performance of (3.14) can be achieved for a prescribed $\ni^2$, if there exist symmetric positive definite matrices P, ψ_k and positive constants a, c such that the following inequalities hold:

$$\Delta_{ij} \equiv [D_{ij}^T P + P D_{ij} + \sum_{k=1}^{L} \psi_k + \sum_{k=1}^{L} P \overline{A}_{iK} \psi_k^{-1} \overline{A}_{ik}^T P + a Y^T Y + a^{-1} P^2 + c^{-1} P^2] + Z < 0$$

for $i = 1, 2, \cdots, \phi$; $j = 1, 2, \cdots, u$; and $k = 1,\ 2,\ \cdots,\ L$. (4.7a)
in which $c = \ni^2$ and $D_{ij} \equiv A_i - B_i F_j$.

Remark 4.2.1: Based on (4.2), the modeling error $\Delta\Phi(t)$ is assumed to be bounded by the specified structured bounding matrix Y and then the larger modeling error results in the larger Y. Hence, the larger modeling error will make Theorem 1 more difficult to be satisfied.

Remark 4.2.2: Eq. (4.7a) can be reformulated into LMI via the following procedure. By introducing new variables $Q = P^{-1}$, $K_j = F_j Q$ and $\overline{\psi}_k = Q \psi_k Q$, Eq. (4.7a) is then rewritten as follows:

$$QA_i^T - K_j^T B_i^T + A_i Q - B_i K_j + \sum_{k=1}^{L} \overline{\psi}_k + \sum_{k=1}^{L} \overline{A}_{iK} \psi_k^{-1} \overline{A}_{ik}^T + aQY^T YQ + a^{-1} I + c^{-1} I + QZQ < 0 \tag{4.7b}$$

for $i = 1,\ 2,\ \cdots,\ \varphi;\ j = 1,\ 2,\ \cdots,\ \mu$; and $k = 1,\ 2,\ \cdots,\ L$. Furthermore, based on Schur's complement [4], [64], it is easy to find that the matrix inequality in Eq. (4.7 b) is equivalent to the following LMI:

$$\begin{bmatrix} \Gamma & YQ & Q & Q & Q & Q \\ (YQ)^T & -(a)^{-1} I & 0 & 0 & 0 & 0 \\ Q & 0 & -(Z)^{-1} & 0 & 0 & 0 \\ Q & 0 & 0 & -(\psi_1)^{-1} & 0 & 0 \\ Q & 0 & 0 & 0 & \ddots & 0 \\ Q & 0 & 0 & 0 & 0 & -(\psi_K)^{-1} \end{bmatrix} < 0 \tag{4.8}$$

for $i = 1,\ 2,\ \cdots,\ \varphi;\ j = 1,\ 2,\ \cdots,\ \mu$; and $k = 1,\ 2,\ \cdots,\ L$

where $\Gamma = QA_i^T - K_j^T B_i^T + A_i Q - B_i K_j + \sum_{k=1}^{L} \overline{A}_{iK} \psi_k^{-1} \overline{A}_{ik}^T + a^{-1} I + c^{-1} I.$

Therefore, Theorem 1 can be transformed into an LMI problem and efficient interior-point algorithms are now available in Matlab LMI Solver to deal with this problem.

Remark 4.2.3 [67]: In order to verify the feasibility of solving the inequalities (4.8) by LMI Solver (Matlab), the interior-point optimization techniques are utilized to compute feasible solutions. Such techniques require that the system of LMI constraints be strictly feasible, that is, the feasible set has a nonempty interior. For feasibility problems, the LMI Solver by feasp♣ is shown as follows:

Find x such that the LMI $\mathrm{L(x)} < 0$ ♦ (4.9a)

as

Minimize t subject to $\mathrm{L(x)} < t \times I$ (4.9b)

♣ feasp is the syntax used to test feasibility of a system of LMIs in MATLAB.

♦ In this study, Eq. (4.9a) can be represented as Eq. (4.8).

From the above, the LMI constraint is always strictly feasible in x, t and the original LMI (4.9a) is feasible if and only if the global minimum $tmin$ of (4.9b) satisfies $tmin < 0$. In other words, if $tmin < 0$ will make Eq. (4.8) be satisfied and then the stability conditions Eq. (4.7a) in Theorem 1 can be met.

Remark 4.2.4 : In order to reduce the computational burden, the positive constants a and c are chosen to be unity in this study.

Based on Theorem 1, we can synthesize a fuzzy controller to stabilize the nonlinear multiple time-delay (NMTD) system. If the designed fuzzy controller cannot stabilize the NMTD system, the fuzzy controller and the dither (as an auxiliary of the fuzzy controller) are simultaneously introduced to stabilize the NMTD system.

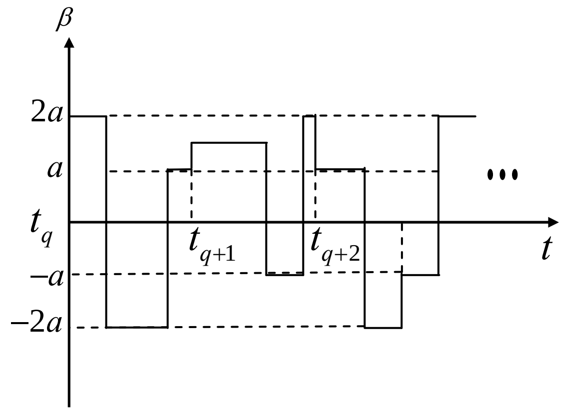
5. NN Relaxed System and Stability Analysis

5.1 Dithered Plant and Relaxed Model

A high frequency signal, commonly called dither $d(t)$, with a finite number η of switching, is injected into the NMTD plant N. Thus, the dithered plant N_d is described as:

$$N_d: \; \dot{X}_d(t) = f(X_d(t),\ U(t), d(t)) + \sum_{k=1}^{L} \rho_k(X_d(t-\tau_k),\ d(t)) + \varpi(t). \quad (5.1a)$$

The algorithm for constructing the dither is given as follows [52]. The time interval [0, T] is divided into an arbitrary number η of equal subintervals. The beginning of the first interval, the end of the first interval, the end of the second interval and the end of the η th interval are denoted by t_0, t_1, t_2 and t_η, respectively. After dividing every interval [t_q, t_{q+1}] for q = 0, 1, 2,…, $\eta - 1$ into ℓ subintervals, the length of the mth subinterval will be $\alpha_m(t_q)[t_{q+1} - t_q]$ for $m = 1,\ 2,\ \cdots,\ \ell$ and the control $\beta_m(t_q)$ is applied at the mth subinterval. Hence, the repetition frequency, shape and amplitude of dither can be determined by regulating the parameters η, $\alpha_m(t_q)$ and $\beta_m(t_q)$. In order to illustrate the algorithm, an example of constructing a dither is given in Fig. 3.



$\alpha_1(t_q) = 0.3,\ \alpha_1(t_{q+1}) = 0.7,\ \alpha_1(t_{q+2}) = 0.4$
$\alpha_2(t_q) = 0.5,\ \alpha_2(t_{q+1}) = 0.2,\ \alpha_2(t_{q+2}) = 0.3$
$\alpha_3(t_q) = 0.2,\ \alpha_3(t_{q+1}) = 0.1,\ \alpha_3(t_{q+2}) = 0.3$

$\beta_1(t_q) = 2a,\ \beta_1(t_{q+1}) = 1.5a,\ \beta_1(t_{q+2}) = a$
$\beta_2(t_q) = -2a,\ \beta_2(t_{q+1}) = -a,\ \beta_2(t_{q+2}) = -2a$
$\beta_3(t_q) = a,\ \beta_3(t_{q+1}) = 2a,\ \beta_3(t_{q+2}) = -a$

Fig. 3. Illustration of constructing a dither.

Remark 5.1.1: According to the above algorithm, the parameters $\alpha_m(t)$ and $\beta_m(t)$ are constant if the dither is chosen to be a periodic signal. Hence, in order to reduce the computational burden, the dither is chosen to be a periodic signal and then $\alpha_m(t)$ and $\beta_m(t)$ are respectively changed to α_m and β_m in the remainder of this study.

The corresponding relaxed model N_r of the dithered plant (5.1a) is defined as [52]:

$$N_r: \; \dot{X}_r(t) = \sum_{m=1}^{\ell} \alpha_m \{ f(X_r(t),\ U(t),\ \beta_m) + \sum_{k=1}^{L} \rho_k(X_r(t-\tau_k),\ \beta_m\} + \varpi(t). \quad (5.1b)$$

in which $\alpha_m(t)$ is non-negative and satisfies the following conditions:

$$0 \le \alpha_m \le 1,\ \sum_{m=1}^{\ell} \alpha_m = 1 \text{ for } m = 1,\ 2,\ \cdots,\ \ell$$

Remark 5.1.2: The curve $X_r(t)$ satisfying (5.1b) is the uniform limit of curves $X_d(t)$ satisfying (5.1a). That is to say, as the frequency of dither goes to infinity, the trajectory $X_d(t)$ described by the dithered plant N_d will approach that of the relaxed model $X_r(t)$ by applying the averaging method to the high-frequency dithered term. Hence, the relaxed model N_r may be viewed as the mathematical model of the NMTD plant N with a dither of high enough frequency.

Based on Remark 5.1.2, it is desired to find the scalar controls α_m and β_m for $m = 1,\ 2,\ \cdots,\ \ell$ such that the trajectories of the relaxed system are UUB. If the trajectories of the relaxed system are UUB and the number η of switching in $d(t)$ is chosen to be sufficiently large, then the dithered plant is approximated by its corresponding mathematical model–the relaxed model and the approximation improves as η increases. Consequently, the trajectory described by the dithered system and that of the relaxed system would be made as close as desired, and then the NMTD system is stabilized.

5.2 NN Relaxed Model

In this subsection, the relaxed model N_r (of the dithered plant N_d) is approximated by an neural-network (NN) model. The procedures of constructing the NN model for N_r are similar to those in Section 3. Therefore, they are not repeated here. The final output of the closed-loop relaxed system $\overline{N_r}$ is described in the following form:

$$\dot{X}_r(t) = \overline{f}_r + \sum_{m=1}^{\ell} \alpha_m \{\sum_{k=1}^{L} \rho_k (X_r(t-\tau_k),\ \beta_m)\} + \varpi(t)$$

$$= \sum_{i=1}^{\varphi} \sum_{j=1}^{\mu} h_i(t) h_j(t) \{D_{ij}(\alpha_m, \beta_m) X_r(t)$$

$$+ \sum_{k=1}^{L} \overline{A}_{ik}(\alpha_m, \beta_m) X_r(t-\tau_k)\} + \varpi(t) + \Delta\Phi_r(t)$$

for $i = 1,\ 2,\ \cdots,\ \varphi;\ j = 1,\ 2,\ \cdots,\ \mu;\ k = 1,\ 2,\ \cdots,\ L;$ and $m = 1,\ 2,\ \cdots,\ \ell$ (5.2)

where

$$\overline{f}_r \equiv \sum_{m=1}^{\ell} \alpha_m \{f(X_r(t),\ U(t),\ \beta_m)\} \text{ with } U(t) = -\sum_{j=1}^{\mu} h_j(t) F_j X_r(t),$$

$$D_{ij}(\alpha_m, \beta_m) = A_i(\alpha_m, \beta_m) - B_i(\alpha_m, \beta_m) F_j\ ,$$

$$\Delta\Phi_r(t) \equiv e_r(t) + \sum_{k=1}^{L} e_{k\,r}(t-\tau_k) \text{ ,in which}$$

$$e_r(t) \equiv \overline{f}_r - \sum_{i=1}^{\varphi} \sum_{j=1}^{\mu} h_i(t) h_j(t) \{D_{ij}(\alpha_m,\ \beta_m) X_r(t)\},$$

$$\sum_{k=1}^{L} e_{k\,r}(t-\tau_k) \equiv \sum_{m=1}^{\ell} \alpha_m \{\sum_{k=1}^{L} \rho_k (X_r(t-\tau_k),\ \beta_m)\}$$

5.3 Stability Analysis of the Closed-Loop Relaxed System

Hereafter, we are concerned with the stability of the closed-loop relaxed system $\overline{N_r}$ instead of discussing that of the closed-loop dithered system $\overline{N_d}$. Hence, the stability criterion of $\overline{N_r}$ is presented in the following.

Theorem 2: The trajectories of the relaxed system $\overline{N_r}$ are UUB and the H^{∞} control performance of (3.14) can be achieved for a prescribed $\ni^2$, if there exist symmetric positive definite matrices P_r, $\psi_{k\,r}$ and positive constants a_r , c_r such that the following inequalities hold:

$$\Delta_{ij}(\alpha_m, \beta_m) \equiv D_{ij}^T(\alpha_m, \beta_m) P_r + P_r D_{ij}(\alpha_m(t), \beta_m) + \sum_{k=1}^{L} \psi_{k\,r}$$

$$+ \sum_{k=1}^{L} P_r \overline{A}_{iK}(\alpha_m, \beta_m) \psi_{k\,r}^{-1} \overline{A}_{ik}^T(\alpha_m, \beta_m) P_r$$

$$+ a_r Y_r^T Y_r + a_r^{-1} P_r^2 + c_r^{-1} P_r^2] + Z_r < 0 \,\dagger,$$

for $i = 1,\ 2,\ \cdots,\ \varphi;\ j = 1,\ 2,\ \cdots,\ \mu$; $k = 1,\ 2,\ \cdots,\ L$; and $m = 1,\ 2,\ \cdots,\ \ell$ (5.3)

in which $c_r = \ni^2$ and

$$D_{ij}(\alpha_m, \beta_m) = A_i(\alpha_m, \beta_m) - B_i(\alpha_m, \beta_m) F_j\,.$$

---

† The representation of Y_r is the same as that of the structured bounding matrix Y in Eq. (4.3).

Proof: The proof of Theorem 2 can be similarly derived by following the same procedure as that in the proof of Theorem 1 but with some extra tuning parameters α_m and β_m. This proof is lengthy, so it is not repeated here.

Remark 5.3.1: By the same procedures as those in Remark 4.2.3, Eq. (5.3) can be rewritten as the following LMIs:

$$\begin{bmatrix} \Gamma_r & Y_r Q_r & Q & Q & Q & Q \\ (Y_r Q_r)^T & -(a_r)^{-1} I & 0 & 0 & 0 & 0 \\ Q & 0 & -(Z_r)^{-1} & 0 & 0 & 0 \\ Q & 0 & 0 & -(\psi_{1r})^{-1} & 0 & 0 \\ Q & 0 & 0 & 0 & \ddots & 0 \\ Q & 0 & 0 & 0 & 0 & -(\psi_{kr})^{-1} \end{bmatrix} < 0 \quad (5.4)$$

for $i = 1, 2, \cdots, \varphi$; $j = 1, 2, \cdots, \mu$; $k = 1, 2, \cdots, L$; and $m = 1, 2, \cdots, \ell$

where

$$\Gamma_r = Q_r A_i^T(\alpha_m, \beta_m) - K_j^T B_i^T(\alpha_m, \beta_m) + A_i(\alpha_m, \beta_m) Q_r - B_i(\alpha_m, \beta_m) K_j + \sum_{k=1}^{L} \overline{A}_{iK}(\alpha_m, \beta_m) \psi_{kr}^{-1} \overline{A}_{ik}^T(\alpha_m, \beta_m) + a_r^{-1} I + c_r^{-1} I.$$

Remark 5.3.2: Similarly, on the basis of Remark 4.2.4, we can solve the inequalities (5.4) via LMI Solver. If $tmin < 0$ will make Eq. (5.4) be satisfied and then the stability conditions Eq. (5.3) in Theorem 2 can be met.

Remark 5.3.3 : In order to reduce the computational burden, the positive constants a_r and c_r are chosen to be unity in this study.

Prior to discussing the stability of the closed-loop dithered system $\overline{N_d}$, stability properties in the finite time interval are defined according to Weiss and Infante [68] as follows.

Definition 5.3.1: A system is stable with respect to the set $\{\rho_1, \rho_2, 0, T, \|x\|\}$, $\rho_1 \le \rho_2$ if for any trajectory $x(t)$ the conditions $\|x(0)\| < \rho_1$, imply $\|x(t)\| < \rho_2$ for $t \in [0, T]$.

Definition 5.3.2: A system is contractively stable with respect to the set $\{\rho_1, \rho_2, \rho_3, 0, T, \|x\|\}$, $\rho_3 < \rho_1 < \rho_2$, if for any trajectory $x(t)$ the conditions $\|x(0)\| < \rho_1$, imply:

(a) stability with respect to $\{\rho_1, \rho_2, 0, T, \|x\|\}$,

(b) there exists $t_1 \in (0, T]$ such that $\|x(t)\| < \rho_3$ for all $t \in [t_1, T]$.

The relaxed system $\overline{N_r}$ may be stabilized by appropriately regulating α_m and β_m. If $\overline{N_r}$ is stable and the number η of switching in $d(t)$ is chosen to be large enough, a high frequency signal (dither) can be constructed through the algorithm proposed by Steinberg and Kadushin [52] for the nonlinear multiple time-delay (NMTD) system $\overline{N}$ such that the dithered system $\overline{N_d}$ is approximated by the relaxed system $\overline{N_r}$ and the approximation improves as η becomes larger. Therefore, the trajectory of $\overline{N_d}$ and that of $\overline{N_r}$ can be made as close as desired. This fact enables a rigorous prediction of stability of $\overline{N_d}$ by establishing stability of $\overline{N_r}$, provided that η is sufficiently large.

Hence, we can deduce the following important theorems.

Theorem 3: The state vector $X_d(t)$ of the dithered system $\overline{N_d}$ is stable with respect to $\{\rho_1, \rho_2, 0, T, \|X\|\}$, if the relaxed system $\overline{N_r}$ is stable in the sense of Lyapunov, provided that η is sufficiently large.

Proof: The algorithm for constructing a dither $d(t)$ given in subsection 5.1 provides a means by which the solutions $X_d(t)$ of the dithered system $\overline{N_d}$ and $X_r(t)$ of the relaxed system $\overline{N_r}$ satisfy

$$\lim_{\eta \to \infty} \|X_d(t) - X_r(t)\| = 0 \cdot$$

Thus, for a certain η we have

$$\|X_d(t) - X_r(t)\| < \varepsilon_1 \quad (5.5)$$

If the relaxed system $\overline{N_r}$ is stable in the sense of Lyapunov, i.e. for each ε_2, it is possible to find a ϑ such that $\|X_r(0)\| < \vartheta$, and we have

$$\|X_r(t)\| < \varepsilon_2 \quad \text{for } t > 0.$$

Thus from (5.5):

$$\|X_d(t)\| = \|X_d(t) - X_r(t) + X_r(t)\|$$
$$\le \|X_d(t) - X_r(t)\| + \|X_r(t)\| < \varepsilon_1 + \varepsilon_2$$
$$\text{for } 0 < t \le T .$$

By taking $\rho_1 = \vartheta$, $\rho_2 = \varepsilon_1 + \varepsilon_2$ stability with respect to $\{\rho_1,\ \rho_2,\ 0,\ T,\ \|X\|\ \}$ is proven.

Theorem 4: The state vector $X_d(t)$ of the dithered system $\overline{N_d}$ is contractively stable with respect to $\{\rho_1,\ \rho_2,\ \rho_3,\ 0,\ T,\ \|X\|\ \}$, if the trajectories of the relaxed system $\overline{N_r}$ are UUB, provided that η is sufficiently large.

Proof: Let the relaxed system $\overline{N_r}$ be UUB. One may select a T large enough so that for a time $t_1 \in (0,\ T]$, we have in addition to the stability properties proven in Theorem 3. According to Definition 3.1, we have that the stability condition of relaxed system is $\|X_r(t)\| < \varepsilon$ for $t \in [t_1,\ T]$.Thus from (5.5):

$$\|X_d(t)\| = \|X_d(t) - X_r(t) + X_r(t)\|$$
$$\le \|X_d(t) - X_r(t)\| + \|X_r(t)\| < \varepsilon_1 + \varepsilon$$
$$\text{for } t_1 < t \le T .$$

Choosing

$\rho_1 = \vartheta$, $\rho_2 = \varepsilon_1 + \varepsilon_2$ and $\rho_3 = \varepsilon + \varepsilon_1$,

it follows that the dithered system $\overline{N_d}$ is contractively stable with respect to $\{\rho_1,\ \rho_2,\ \rho_3,\ 0,\ T,\ \|X\|\ \}$.

6. CONCLUSIONS

This study presents an effective approach to stabilize the nonlinear multiple time-delay (NMTD) systems by fuzzy controllers and dithers. The fuzzy controller and the dither are simultaneously introduced to stabilize the NMTD system. Simulation results demonstrate that the fuzzy controller can stabilize the NMTD system by appropriately regulating the parameters of dither when the dither's frequency is high enough.

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