

# Use the Modified STAR 16-QAM Constellation Extension Scheme to Improve Peak to Average Power Ratio of Single Carrier Frequency Division Multiple Access System

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## Abstract

Since the peak to average power ratio (PAPR) of Single Carrier Frequency Division Multiple Access (SC-FDMA) system is easily increased by the changes of selected modulation and pulse shaping technology, hence, how to improve the PAPR efficiency of SC-FDMA system further becomes one of the hottest study topics now. This paper proposes a kind of new star 16-QAM constellation extension scheme (CES) to improve PAPR efficiency of SC-FDMA system. The simulation results indicate that the recommended method can integrate modulation and PAPR reduction technology as well as having better PAPR efficiency than traditional SC-FDMA system.

**Keywords:** SC-FDMA, PAPR, CES and star QAM.

## 1. Preface

Long Term Evolution (LTE) standards are the early technology of the 4<sup>th</sup> mobile communications technology standards, and are proposed by the 3rd Generation Partnership Project (3GPP), wherein Long Term Evolution Advanced (LTE-Advanced) is recognized as the 4<sup>th</sup> mobile communication technology standards by International Telecommunication Union (ITU). In Long Term Evolution Advanced standards, the uplink and downlink transmission techniques are techniques adopted Orthogonal Frequency Division Multiple Access (OFDMA) and Single Carrier Frequency Division Multiple Access (SC-FDMA) respectively. The two techniques are achieved on the basis of the structure of Orthogonal Frequency Division Multiplexing (OFDM) technology [1-4].

Though the SC-FDMA technology has received wider attention by all circles, it still has much

improvement space, such as peak to average power ratio (PAPR) of SC-FDMA is easily increased by the changes of selected modulation and pulse shaping technology. Hence, how to improve the PAPR efficiency of SC-FDMA system further becomes one of the hottest study topics now. [5-9]

Currently, proposed methods to improve the PAPR of OFDM are pretty many, such as selective mapping (SLM) [10], partial transmit sequence, (PTS) [11], Tone Reservation (TR) [12] and constellation extension scheme (CES) [13] etc. However, above improvement methods cannot be applied on SC-FDMA technology directly. The two techniques of SC-FDMA and OFDM only have differences on input signals, which are time domain and frequency domain signals respectively, and SC-FDMA technology has different PAPR efficiency due to different subcarrier mapping ways, such as localized FDMA (LFDMA), interleave FDMA (IFDMA) and other mapping ways. Therefore, this paper starts studying a new type of method to reduce PAPR which can be applied on SC-FDMA technology. On the other aspect, quadrature amplitude modulation (QAM) technology is a digital modulation technology which is used widely currently, wherein square QAM modulation technology is applied on various communication systems most. However, compared with star QAM modulation technology, the efficiency of square QAM modulation technology may be affected by the performance of coherent detection. Hence, the star QAM modulation is focused more [14]. This paper will study a kind of method which can combine star QAM modulation technology and reduce peak to average power ratio technology simultaneously, and can be applied on 16-QAM modulation Single Carrier Frequency Division Multiple Access system.

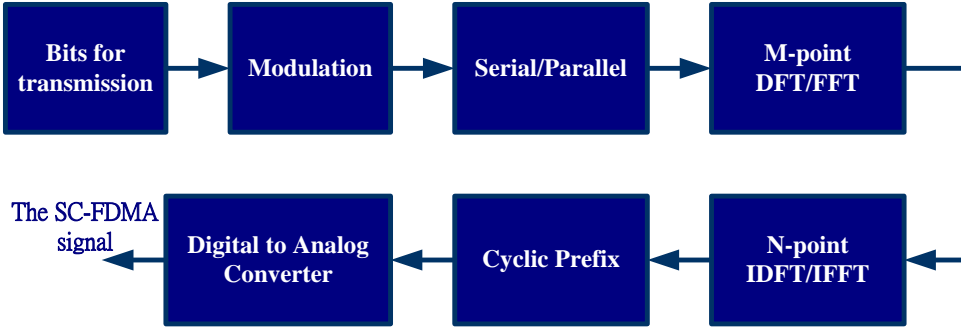


Fig. 1 Block Diagram of Transmission Terminal of SC-FDMA System

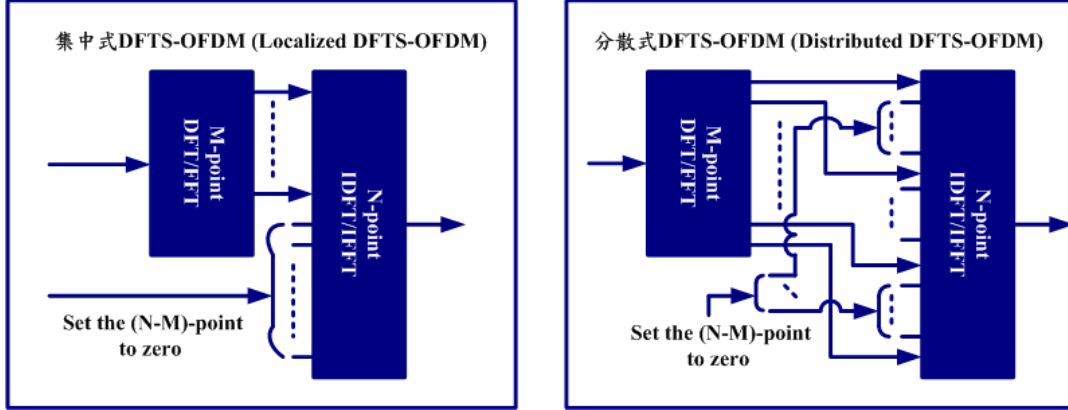


Fig. 2 Distributed DFTS-OFDM and Localized DFTS-OFDM

## 2. SC-FDMA System and PAPR

In Fig. 1, material transmission will be processed via modulation. The modulation technologies include M-ary Phase Shift Keying (PSK) or M-ary QAM. Then the modulated results will be converted into frequency domain signal through Discrete Fourier Transform (DFT) of M-point. Then the frequency domain signal is delivered to inverse DFT of N-point and converted into SC-FDMA signal.

See Fig. 2 on the placement method of input materials of inverse DFT. Generally, it is divided into Distributed and localized types. They are called distributed discrete Fourier transform spreading orthogonal frequency division multiplexing (Distributed DFTS-OFDM) and localized discrete Fourier transform spreading orthogonal

frequency division multiplexing (Localized DFTS-OFDM). Distributed DFTS-OFDM is to place input materials of inverse discrete Fourier transform according to the set interval. If the interval is a fixed value, the Distributed DFTS-OFDM is also called interleaved discrete Fourier transforms spreading orthogonal frequency division multiplexing (Interleaved DFTS-OFDM). Localized DFTS-OFDM is to place the input material in the input terminal of inverse discrete Fourier transform continuously. For example, to express the  $M$  symbols to be transmitted as  $\mathbf{a} = [a_1, a_2, \dots, a_M]$ , the results can be generated after discrete Fourier transform calculation of  $M$ -point and can be written as

$$X_k = \sum_{i=0}^{M-1} a_i e^{-j \frac{2\pi i k}{M}}, k = 0, 1, \dots, M-1.$$

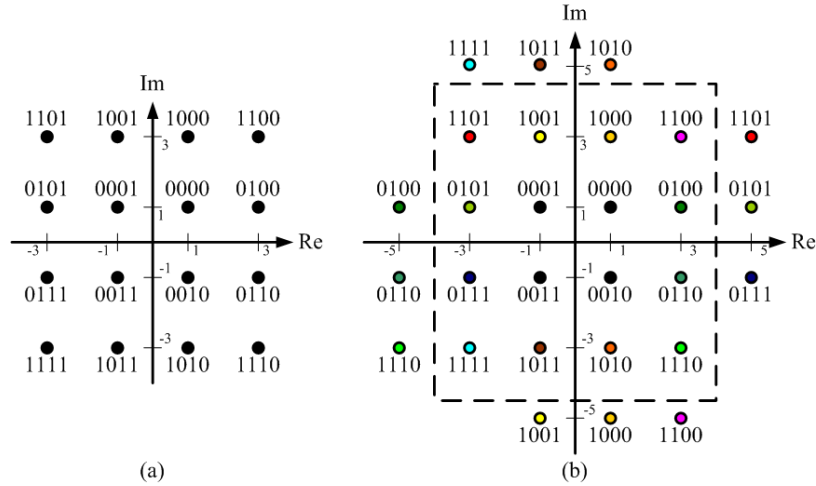


Fig. 3: (a) 16-QAM Signal Constellation Diagram and (b) 16-QAM Extension Constellation Diagram Adopting Gray Mapping

Supposed Localized DFTS-OFDM technology is used, and then the input materials of inverse discrete Fourier transform of  $N$ -point can be expressed as

$$\bar{X}_p = \begin{cases} X_p, & p = 0, 1, \dots, M-1 \\ 0, & p = M, M+1, \dots, N-1 \end{cases} \quad (1)$$

If Distributed DFTS-OFDM technology is used and the interval number of input materials is  $Q$ , the following examples can be used,

$$\bar{X}_p = \begin{cases} X_p, & p = 0, Q, 2Q, \dots, MQ \\ 0, & p \neq 0, Q, 2Q, \dots, MQ \end{cases} \quad (2)$$

wherein  $QM \leq N$ . To simplify the names conveniently, the paper also calls the Localized DFTS-OFDM as single carrier localized frequency division multiple access (SC-LFDMA), and Interleaved DFTS-OFDM as single carrier interleaved frequency division multiple access. Then, the generated DFTS-OFDM signal after inverse DFT calculation of  $N$ -point can be expressed as the following:

$$x_s = \frac{1}{N} \sum_{p=0}^{N-1} \bar{X}_p e^{\frac{j2\pi sp}{N}}, \quad s = 0, 1, \dots, N-1 \quad (3)$$

The definition of the PAPR of the DFTS-OFDM signal can be expressed as the following:

$$\text{PAPR} = 10 \log_{10} \left( \frac{\max_s |\bar{x}_s|^2}{E\{|\bar{x}_s|^2\}} \right), \quad s = 0, 1, \dots, N$$

wherein  $E\{u\}$  is the expected value of  $u$ . In other words, the PAPR of any signal is defined as the

ratio of max instantaneous power of the signal to its mean power. Generally, complementary cumulative distribution function (CCDF) is used to analyze and assess the efficiency reduction of PAPR. Supposed the standard reference value of PAPR is  $\text{PAPR}_0$ , CCDF (PAPR) is the probability that the PAPR of transmission signal is larger than  $\text{PAPR}_0$  and it can be expressed as

$$\text{CCDF}(\text{PAPR}) = P_r(\text{PAPR} > \text{PAPR}_0)$$

### 3. Constellation Extension Scheme (CES)

Constellation Extension Scheme (CES) is mainly to change the phase of transmission signal through the modulation program, and multiple candidate signals are generated on the basis of it to express the originally transmitted materials. The signals to be transmitted are expressed by the signal with the smallest PAPR, which are selected from the candidate signals. Firstly, the CES is to divide the constellation points in the signal constellation diagram into two sets to form extension constellation diagram. The two sets are constellation points set without extension constellation points and constellation points set with extension constellation points. The CES is to take modulation for the input materials bits content according to the extension constellation diagram, and change the constellation points with extension into extension constellation points to derive multiple candidate signals further for transmission signals selection.

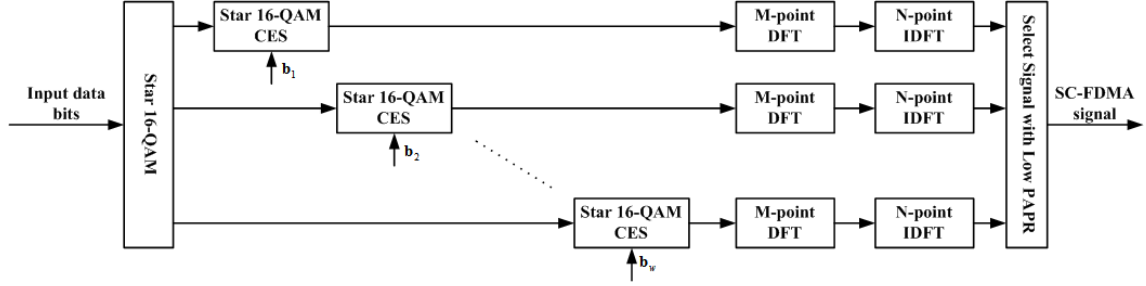


Fig. 4 Block Diagram with combination of Extension Constellation Scheme and Selection Mapping to 16-QAM Modulation Single Carrier Frequency Division Multiple Access System

The candidate signal with the smallest PAPR is selected as the transmission signals. Take the following 16-QAM CES as an example for illustration. The 16-QAM signal constellation diagram using Gray mapping and its extension constellation diagram are indicated in Fig 3. In Fig 3(a), each constellation point of 16-QAM signal constellation diagram has the only mapped material bit content. In contrast, 16-QAM extension constellation diagram in Fig 3(b) means partial constellation points have the same mapped materials bits content. Supposed 16-QAM extension constellation diagram is based on that the decimal of digital content of input materials is no less than 4, the mapped constellation points have extension constellation points, as in Fig 3(b). For example, when the bit content of input materials is {1111}, (i.e. the decimal value is 15), the mapped constellation point of the bit content includes another extension constellation point  $\{-3+j5\}$  besides the existed  $\{-3-j3\}$ .

Supposed totally  $s$  constellation points of the input materials have extension constellation points with modulation, (i.e. corresponding decimal values of digital contents of constellation points are all no less than 4), and max extension constellation point number of each constellation point is 1, it can generate  $2^s$  candidate signals for the transmission terminal to select the signals with the smallest peak to average power ratio as the transmission signals. However, when the  $s$  number is rather large, the number of candidate signals will be in Exponential Growth and the circuits with extension constellation scheme is difficult to be achieved. Therefore, it is an important research direction on how to improve selection mechanism, so that the extension constellation mechanism has less calculation and better peak to average power ratio efficiency.

#### 4. Recommended Method

Fig. 4 is the block diagram with combination of CES and SLM to 16-QAM modulation SC-FDMA system. For the configuration of star 16-QAM

signal constellation diagram, the plan will design the star 16-QAM extension constellation diagram configuration to obtain the optimal PAPR on the basis of measurement of minimum Euclidean distance, minimum phase rotation angle and PAPR. Fig. 5 is the star 16-QAM extension constellation diagrams with two kinds of different configurations studied in this paper. First, this paper makes the input material bit string

$$\mathbf{c} = \{c_{i,j} | i = 1, 2, \dots, M \text{ and } j = 1, 2, 3, 4\}$$

Takes modulation according to the star 16-QAM extension signal constellation diagram indicated in Fig 5. Then, randomly generated  $W$  bit sequences with length of  $M$  are taken as the phase disrupting factor of selection mapping

$$\mathbf{b}_i = [b_{i,1}, b_{i,2}, \dots, b_{i,M}], i = 1, 2, \dots, W,$$

wherein  $\mathbf{b}_1$  is the digital sequence with content of 0.  $W$  candidate sequences determine whether the symbols vector is taken extension constellation scheme through the phase disrupting factor, i.e. when  $b_{i,j} = 0$ , the  $j$ th constellation point is not changed into extension constellation point. When  $b_{i,j} = 1$  the  $j$ th constellation point is changed into extension constellation point.  $W$  candidate sequences are generated by transmitting the  $W$  disrupted modulation signals to DFT of  $M$ -point and inverse DFT of  $N$ -Point. SC-FDMA signals to be transmitted are selected from the  $W$  candidate signals with the smallest PAPR as the transmission signals. Since the contents of  $W$  candidate sequences are in the extension constellation diagram, hence, additional information is unnecessary to be transmitted from the transmission terminal to the reception terminal, and it can improve the shortcoming of that original SLM technology needs additional information transmission. Additionally, based on the SLM technology that generates phase disrupting, it can simplify a large number of computational complexities required by CES technology.

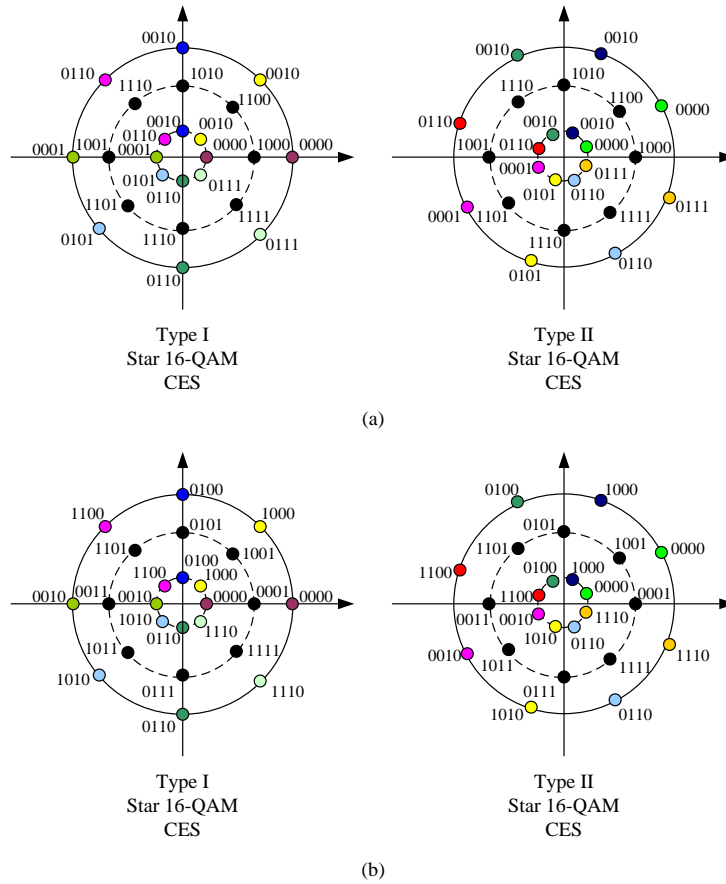


Fig. 5 (a) Star 16-QAM Extension Signal Constellation Diagram by Two Kinds of Different based on Bit Mapping Method  
 (b) Star 16-QAM Extension Signal Constellation Diagram by Two Kinds of Different based on Setting Division Mapping Method

To evaluate the PAPR improvement efficiency of recommended method applied on SC-FDMA system effectively, the author considers different bit mapping methods to take evaluation for recommended methods, wherein  $M=256$  and  $N=512$  for DFT of  $M$ -point and inverse DFT of  $N$ -Point. In simulation results, the input materials take randomly generated 10,000 OFDM materials blocks for simulation. To calculate PAPR values of all signals accurately, the simulation results adopt 4 times of oversampling for simulation. For number selection of  $W$  candidate sequences,  $W=2,4,8$ . Fig. 6 is to compare CCDF of PAPR by applying different bit mapping methods and constellation diagrams configuration on SC-FDMA system. In Fig. 6 and Fig. 7, selection mapping technology is used in Localized DFTS-OFDM and Interleaved DFTS-OFDM and optimal PAPR efficiency can be obtained. Though the PAPR efficiencies of recommended type I and type II bit mapping methods not better than SLM

technology, the recommended methods do not need to transmit information  $\lceil \log_2 W \rceil$  bit to reception terminal additionally for original information transmission use. Furthermore, the PAPR efficiencies of recommended methods are also better than Localized DFTS-OFDM and Interleaved DFTS-OFDM.

## 5. Conclusion

This paper takes modulation technology based on star QAM as the main modulation technology and integrates PAPR reduction technology of extension constellation scheme and selection mapping to be applied on SC-FDMA system. Since the contents of candidate sequences if recommended methods are all in the extension constellation diagram, the transmission terminal is unnecessary to transmit additional information to the reception terminal, and it can improve the shortcoming of that original selection mapping technology needs

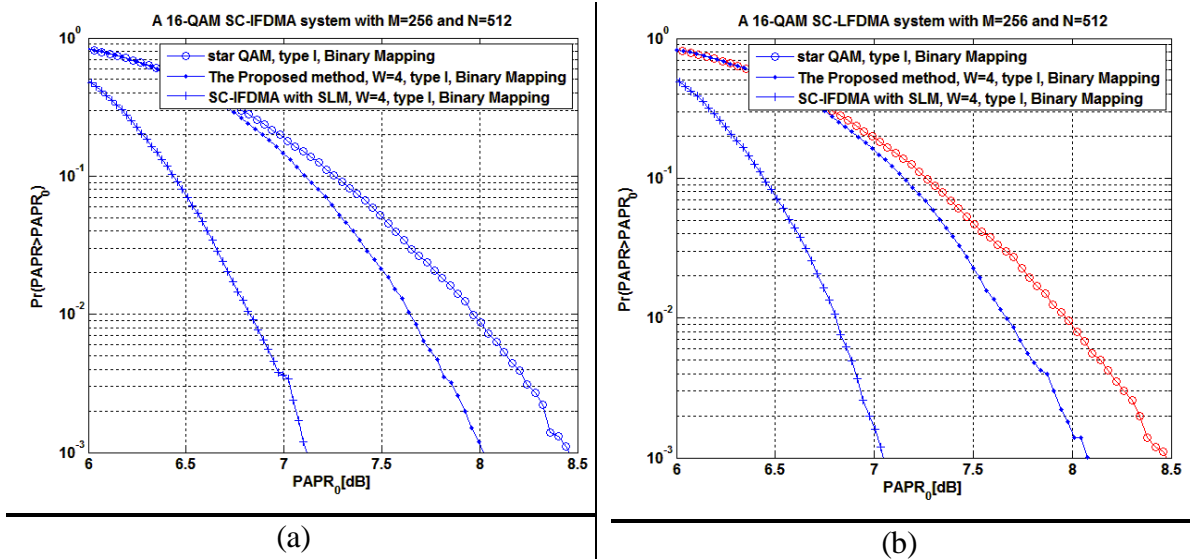


Fig. 6 Complementary cumulative distribution function figure of peak to average power ratio by applying type 1 bit mapping method on Single Carrier Frequency Division Multiple Access system (a) SC-IFDMA (b) SC-LFDMA

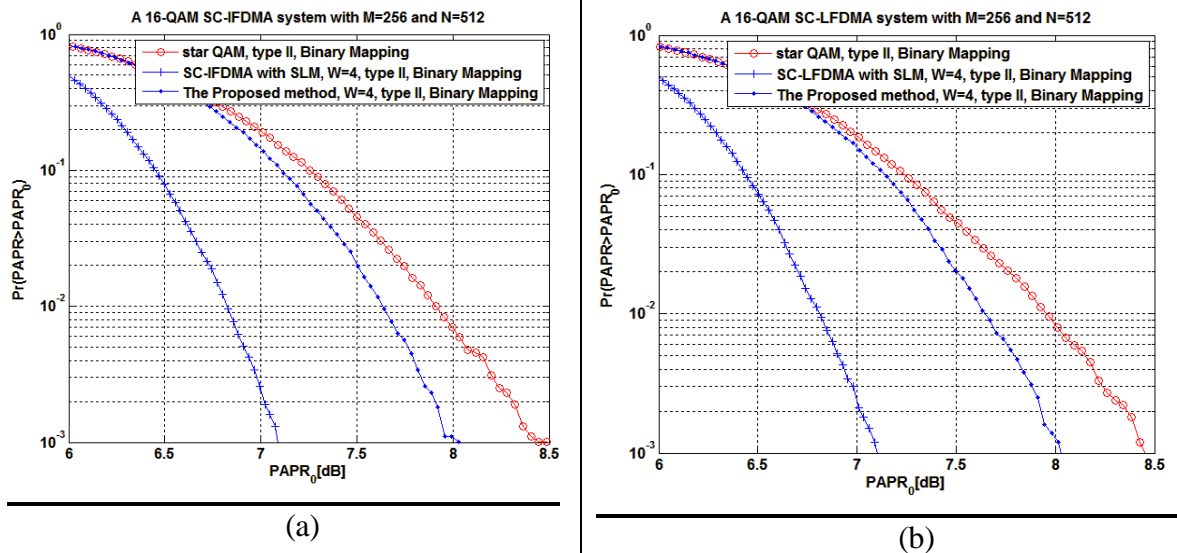


Fig. 7 Complementary cumulative distribution function figure of peak to average power ratio by applying type 2 bit mapping method on Single Carrier Frequency Division Multiple Access system (a) SC-IFDMA (b) SC-LFDMA

additional information transmission. Additionally, based on the selection mapping technology that generates phase disrupting, the recommended methods can simplify a large number of computational complexities required by extension constellation scheme technology.

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